Physics Based Approaches to HRTF Computation and Understanding

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About Myself

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- Worked in the field of Linear and Nonlinear Acoustics, Fluid Mechanics, Applied Mathematics in Russian Academy of Sciences and Universities in Russia and R&D companies in the US;
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Why we should model Human Hearing?

- Understanding of Mechanisms of Localization of Sound Sources and Orientation in Space;
- Generation of Virtual Audio Reality;
- Various Practiccal Applications:
  - Quality Sound Devices
  - Audio/Video Tracking of Sound Sources
  - Speech Recognition
  - Blind Orientation

Introduction

Simple Short Wave (Ray) Model

Major Physical Effects:
1. Reflection/Absorption by the boundaries;
2. Diffraction.

Effect of Scattering

Scattering Neglected
Scattering Taken into Account

\( ka = 1 \)
\( ka = 5 \)
\( ka = 10 \)

(\( k \) is the wavenumber, \( a \) is the radius of the sphere)
Head Related Transfer Function (HRTF)

What does HRTF depend from?
- Size and shape of the head, pinnae, and body;
- Hair, beard, mustache;
- Distribution of sound sources in the room;
- Architecture (size and shape of the room, furniture, material of walls, floors, ceiling, carpets);
- Position and orientation of the head;
- Ambient conditions (temperature, humidity, pressure) (?)
- Other factors (?)

Example of HRTF for Sphere in Plane Acoustic Wave

Wave Equation
\[ \frac{\partial^2 p}{\partial t^2} - c^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right) = \epsilon \nabla^2 p \]

Fourier Transform from Time to Frequency Domain
\[ p'(x, y, z, t) = \int P(x, y, z, \omega) e^{-i\omega t} d\omega \]

Helmholtz Equation
\[ \nabla^2 P + k^2 P = 0, \quad k = \omega/c \]

Boundary Conditions for the Helmholtz Equation
- Sound-hard boundaries: \( \frac{\partial P}{\partial n} = 0 \)
- Sound-soft boundaries: \( P = 0 \)
- Impedance boundary conditions: \( \frac{\partial P}{\partial n} + i\sigma P = 0 \)
- Sommerfeld’s radiation condition (for infinite domains): \( \lim_{r \to \infty} \frac{\partial P}{\partial n} + i\sigma P = 0 \)

Numerical Methods
- Boundary Element Method;
- Spectral Methods;
- Finite Difference, Finite Element, and other methods requiring spatial meshing;
- Meshless Methods
Boundary Element Method (BEM)

Green's Function for Helmholtz Equation:
\[ G(x,y) = \frac{1}{4\pi|x-y|} \]

Green's Identity for Helmholtz Equation:
\[
\alpha P(x) = \int \left( P(y) \frac{\partial G(x,y)}{\partial n} - G(x,y) \frac{\partial P(y)}{\partial n} \right) dS(y) + QG(x) \]

Methods of Solution

Analytical Methods

- Relatively simple geometry (sphere, ellipsoid, some symmetries, planar) -- Expansions in series of orthogonal eigenfunctions of the Helmholtz Equation:
  - Duda & Matrens, 1999; Hanish, 1981; Morse & Feshbach, 1953;
- Perturbations of simple geometry -- Asymptotic techniques;
  - Modifying spherical head solution to actual head shape;
- Short Wave and Long Wave Approximations -- Asymptotic techniques:
  - Kirchhoff, Rytov, Born, Fraunhofer, Fresnel;
- Modeling of Boundary Conditions -- Acoustics of Complex Media.

Some Problems To Solve

Reflection from Walls (method of images)
HRTF for Sphere with Impedance Boundary Conditions

May depend on frequency!

\[ \frac{\partial P}{\partial t} + \nabla \cdot \mathbf{P} = 0 \]

\( \mathbf{P} = \mathbf{P} \delta \) \( \text{at} \) \( \partial \Omega \)

Effect of Hair on HRTF

Katz (1998) shows that the effect of skin impedance can be neglected, while the effect of hair on HRTF can be 7dB. This should be checked out.

Hair covers only a part of the head. The model should take into account changes of the acoustic impedance over the head.

Sensitivity Analysis of HRTF to Slight Changes of the Domain Boundary

\[ \epsilon = a \left[ 1 + \epsilon F(\theta, \phi) \right], \ \epsilon \ll 1, \]

\[ P(\theta, \phi) = \sum_{m,n} B_{mn} \mathbf{h}_n(\sin \theta) Y_m^0(\theta, \phi) + \text{Sources}, \]

Reexpand the surface function into a series of orthogonal spherical harmonics to determine \( A_{mn} \).

Conclusions

- Multiple physical effects on HRTF should be evaluated;
- There exist analytical and numerical methods to compute accurately HRTF;
- Numerical and analytical methods complement each other and should be used both to verify solutions, evaluate accuracy, and obtain insight to the problem;
- Experimental validation is important;
- There is a problem of proper parametric study.

Boundary Element Software

- \text{COMET Acoustics}
- Galerkin and collocation based formulation
- Solver is a direct one
- Tasks
  - Determine limits
  - Improve using iterative solution techniques

Thank You for Attention!