EFFECTIVE CIRCLE-CONSTRUCTION ALGORITHMS FOR MINIMIZING THE WAVELENGTH REQUIREMENT IN WDM RINGS

Guangyu Zhu,¹ Fei Yuan,¹ and H. Ghafouri-Shiraz¹
¹ School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore 639798

Received 22 February 2001

ABSTRACT: Circle constructions can find many applications in the system design phase of WDM ring networks to optimize system performance, while still efficiently utilizing the network resources under consideration (e.g., total number of wavelengths or transceivers). In this letter, we present a series of matrix-based circle-construction algorithms to minimize the number of required wavelengths in both unidirectional and bidirectional rings under an arbitrary traffic pattern. A Matlab implementation of the algorithms can effectively generate the optimum design solution in terms of the total cost on wavelengths. Thus, it ensures that the channel bandwidth and I/O capacity in the WDM rings are utilized most efficiently. © 2001 John Wiley & Sons, Inc. Microwave Opt Technol Lett 30:221–225, 2001.

Key words: circle construction; wavelength-division multiplexing (WDM) rings; uniform traffic; distance-dependent traffic

1. INTRODUCTION

There is a great deal of interest in designing networks deploying wavelength-division multiplexing (WDM) SONET/SDH ring due to their large capacity and reliability. In practice, the first steps in designing such ring networks usually involve laying out the topology and routing the network traffic. However, as the number of nodes in the ring $N$ exceeds ten, it is generally a challenging job for network designers to assign the connections of the given traffic pattern to different wavelength channels, while still utilizing the channel bandwidth and I/O capacity in the most efficient manner.

In this letter, we investigate generic cost-effective design techniques to support a given (static) all-to-all traffic pattern in both unidirectional and bidirectional WDM rings with an arbitrary number of nodes. Two algorithms, namely, the full circle-construction algorithm (FCCA) and the partial circle-construction algorithm (PCCA), are proposed and implemented to minimize the number of required wavelengths with uniform and nonuniform traffic. In our analysis, we also consider the fact that the overall network cost includes the cost of the transceivers required at each node, as well as the number of wavelengths. As pointed out in [1–3], the transceiver cost normally includes the cost of terminating equipment, as well as higher layer electronic multiplexing equipment, which in practice can dominate over the cost of the wavelengths in the WDM ring network. However, as shown in a companion paper [4], the circle-construction techniques proposed can be used together with proper wavelength-assignment algorithms to effectively reduce the total cost on transceivers. In the case in which each node only has a limited number of transceivers, Zhang and Qiao [5] have proved that circle constructions can actually become a part of the scheduling algorithms for achieving optimum network throughput.

2. THEORETICAL ANALYSIS

The problem of assigning network connections to wavelength channels in a WDM ring can be perfectly modeled by constructing circles. Each circle has an appropriate number of drops, and one full circle represents a wavelength channel. Assume that the traffic grooming factor is $g$; each wavelength channel carries exactly $1/g$ of the capacity of a wavelength. If the total number of circles (or channels) needed to carry a given traffic pattern is $C$, the number of required wavelengths $W$ is $C/g$. That is, the minimum number of required wavelengths occurs if and only if all of the connections can be assigned properly to the minimum number of channels.

For better illustration, we number the nodes in the WDM ring from 1 to $N$. All-to-all traffic requires a connection between every source and destination nodes. This implies a total of $N(N-1)$ connections (half of these are in the clockwise direction, and the other half are in the counterclockwise direction) on the fiber. Each of these connections may carry a wavelength or subwavelength of traffic. In the following discussions, we examine circle-construction techniques in unidirectional and bidirectional rings of arbitrary size with uniform traffic. In Section 3, we analyze how these techniques can be applied in the full circle-construction algorithm (FCCA) to minimize $W$ with an arbitrary nonuniform traffic.

In unidirectional rings with uniform traffic, a total of $C = N(N - 1)/2$ circles are needed. One simple way to construct circles is to group two connections sharing the same wavelength. In this case, all-to-all traffic requires a connection between every source and destination nodes, as shown in Figure 1(a). In bidirectional rings, we assume that the traffic is always routed over the shortest path. More specifically, traffic is carried to at most $(N - 1)/2$ (when $N$ is odd) and $N/2$ (when $N$ is even) hops away in either direction. It was shown in [6] that the number of circles $C$ required to carry uniform all-to-all traffic in bidirectional rings is given by

$$C = \begin{cases} \frac{N^2 - 1}{8}, & \text{for } N \text{ odd} \\ \frac{N^2}{8}, & \text{for } N \text{ even.} \end{cases} \quad (1)$$

There are a total of $N(N - 1)/2$ drops that need to be assigned to these circles. Thus, the average number of drops per circle $\overline{M}$ is

$$\overline{M} = \begin{cases} \frac{N(N - 1)}{2} \cdot \frac{8}{N^2 - 1} - \frac{4N}{N + 1}, & \text{for } N \text{ odd} \\ \frac{N(N - 1)}{2} \cdot \frac{8}{N^2} = \frac{4(N - 1)}{N}, & \text{for } N \text{ even.} \end{cases} \quad (2)$$

As Eq. (2) indicates, in both cases, $\overline{M}$ is less than 4, and approaches 4 as the number of nodes $N$ increases. In fact, for an arbitrary value of $N$, we are able to construct circles with up to four drops to support all-to-all uniform traffic. For example, for $N = odd$, we have

$$\begin{cases} \frac{(N - 1)(N - 3)}{8} + \frac{8}{2} = \frac{N^2 - 1}{8} \\ 4 \cdot \frac{(N - 1)(N - 3)}{8} + 3 \cdot \frac{(N - 1)}{2} = \frac{N(N - 1)}{2}. \end{cases} \quad (3)$$
This means that we can always construct \((N - 1)(N - 3)/8\) four-drop circles and \((N - 1)/2\) three-drop circles to connect every single source–destination pair in a given direction. Similar results hold when \(N\) is even. The only difference is that we need to form appropriate numbers of four-drop and two-drop circles in this case. As shown in Table 1, the expressions for the numbers of four-drop and two-drop circles differ somehow, depending on whether \(N = 4m\) or \(4m + 2\) for some integer \(m\). Zhang and Qiao [5] proposed two circle-construction methods for bidirectional rings with even and odd number of nodes, respectively, namely, complementary assembling with dual strides (CADS) and complementary assembling with triadic strides (CATS). As illustrated in Figure 1(b), (c), CADS combines either two or four connections, while CATS combines either three or four connections to form full circles. A two-drop circle shown in Figure 1(b) is actually a special case of a four-drop circle, with two drops overlapping with the other two (i.e., when \(s = N/2\)). Similarly, as illustrated in Figure 1(c), a three-drop circle occurs when the fourth drop overlaps with the first one (i.e., when \(s = 0\)). The solid lines and dotted lines denote the circles constructed in the clockwise and counterclockwise directions, respectively. The validity of CADS and CATS algorithms is not difficult to prove. Here, we just provide a simple proof for CATS. The end nodes in CATS algorithm are generally expressed as \(i\), \([i + (N - 1)/2 - s]\), \([i + (N - 1)/2 - s]\), and \((i - s)\). For each starting node \(i\), \(s\) varies from \((i - 1)\) to \(0\). Thus, the total number of unique circles constructed is \(\sum_{i=1}^{(N-1)/2} i = (N^2 - 1)/8\), which is equal to the minimum number of circles given in Eq. (1) when \(N\) is odd.

Note that, even if the combination of four-drop and three-drop (or two-drop) circles is fixed (as shown in Table 1), potentially, a large number of actual circle patterns with that combination exist due to the symmetry in the ring. To be consistent in our following discussion, we only illustrate using the circles generated by CATS (for \(N\) odd) and by CADS (for \(N\) even). Figures 2 and 3 show the circles constructed using CADS (for \(N = 8\)) and CATS (for \(N = 9\)), respectively. The end nodes in two-drop and three-drop circles are represented by squares in these diagrams, while those in four-drop circles are indicated by solid circles. A two-drop circle may be considered shared in both directions because, when \(N\) is even, the distances of the two paths in the clockwise and counterclockwise directions are the same. Theoretically, the traffic may be routed in either direction from the source to the destination, which is exactly half a ring apart. To avoid this ambiguity, we may distribute half of the two-drop circles to each direction (when \(N = 4m\)). However, when \(N = 4m + 2\), one direction will end up with one more circle than the other direction as the total number of circles in both directions, \(N^2/4\) is odd.

### Table 1: General Expressions for the Number of Circles Constructed in Bidirectional Rings

<table>
<thead>
<tr>
<th>Case</th>
<th>Four-Drop Circles</th>
<th>Two-Drop Circles</th>
<th>Four-Drop Circles</th>
<th>Three-Drop Circles</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N = 4m)</td>
<td>(N(N - 2)/8)</td>
<td>(N/4)</td>
<td>((N - 1)(N - 3)/8)</td>
<td>((N - 1)/2)</td>
</tr>
<tr>
<td>(N = 4m + 2)</td>
<td>(N(N - 2)/8)</td>
<td>((N - 2)/4)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 2](image2.png)  
*Figure 2: Circles constructed using CADS in a bidirectional ring with eight nodes*
3. ALGORITHMS FOR MINIMIZING WAVELENGTHS

For uniform all-to-all traffic, the circle-construction methods examined in Section 2 can assign all of the connections to the minimum number of circles, thus effectively yielding the minimum number of wavelengths. In this section, we propose two algorithms to minimize the total number of wavelengths for nonuniform traffic, in which the units of traffic between different source–destination pair are generally different. The simulation program implemented from these two algorithms can automatically process an arbitrary nonuniform traffic pattern, and calculate the cost-effective circle-construction design of a WDM ring network using the minimum wavelengths. The core of the program algorithm is outlined in Algorithm I in pseudocodes. The idea here is to divide circle constructions into two stages, each governed by a different algorithm. In stage 1, we use the full circle-construction algorithm (FCCA) to form as many full circles \( C_i \) as possible using the set of circle patterns generated by the circle-construction techniques. Then, another algorithm, the partial circle construction algorithm (PCCA), is used to distribute the remaining connections into a minimum number of circles \( C_i \). Consequently, the minimum number of total circles \( C = C_1 + C_2 \) are formed to support the given traffic pattern. That is, a minimum of \( C/g \) wavelengths is required in the WDM ring for the given nonuniform traffic. Although the general idea to divide the circle constructions into two stages was first suggested in [8], to the best of our knowledge, this letter is the first that introduces matrix-based circle-construction techniques.

To facilitate computer processing, we first transform a given nonuniform traffic pattern into its matrix form \( G_{i,j} \). The element \( G_{i,j} \) in the matrix is the traffic unit(s) from the source node \( i \) to the destination node, which is \( s \) hops away along the designated path (i.e., the shortest path in bidirectional rings). In this representation, a unidirectional ring has a traffic matrix of dimension \( N \times (N - 1) \). The dimension of the traffic matrix of a bidirectional ring with an odd number of nodes \( N \) is \( N \times (N - 1)/2 \), while for \( N \) even, it is \( N \times N/2 \). Also, we need to transform the circles generated by the circle-construction techniques into proper matrix form. In this case, we write a 1 on row \( i \) and column \( (i + s) \) in its circle matrix (i.e., \( G_{i,i} = 1 \), if a circle contains a connection from node \( i \) to node \( (i + s) \). In other words, the circle matrices based on CATS have exactly three or four 1s, and the rest of the matrix elements are all 0. Since each individual connection is grouped into a unique circle in circle construction, none of the circle matrices contains nonzero elements in the same position of the matrix.

The full circle-construction algorithm (FCCA) aims to construct as many full circles \( C_i \) as possible by simple matrix subtraction. The traffic matrix is subtracted by each individual circle matrix until one or more common elements of them in the traffic matrix reduce to zero. Since none of the circle matrices contains overlapping elements, the sequence of matrix subtraction does not matter. After each successful subtraction, a full circle is formed according to the pattern indicated by the corresponding circle matrix. Then we use the partial circle-construction algorithm (PCCA) to process the remaining traffic matrix \( G_{i,j} \). The goal of the PCCA is to form the minimum number of full or partial circles \( C_j \) from \( G_{i,j} \) by considering the connections with the longer strides first. A connection is fitted into an existing circle as long as it does not overlap with any of the existing connections in that circle. In other words, a new circle is created for a connection only if there is no vacancy in any existing circles. Let us examine the entire process of these steps from an example below. Suppose the given traffic matrix for a five-node ring is \( G_{i,j} \). After subtracting the three-circle matrix \( C_i = 1 \), \( C_i^2 \), and \( C_i^3 \) (as indicated by circles 1–3 in Fig. 4) from \( G_{i,j} \), four full circles \( C_i = 4 \) are formed. The remaining matrix is then processed by the subroutine based on the PCCA. Another two circles \( C_j = 2 \) are formed in this round. One of them is a full circle, and the other one is a partial circle.

\[
\begin{align*}
G_{i,j} = & \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 2 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}, & 
C_{i,j}^1 = & \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \\
C_{i,j}^2 = & \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, & 
C_{i,j}^3 = & \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{align*}
\]
By FCCA, \( \{G_{i,j}\} = 2 \cdot \{C_{i,j}^1\} + \{C_{i,j}^2\} + \{C_{i,j}^3\} \).

By PCCA, \( \{G_{i,j}\} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \).

\[ 4. \text{ RESULTS AND DISCUSSIONS} \]

Algorithm I provides a systematic approach to minimizing the number of required wavelengths for an arbitrary traffic pattern in both unidirectional and bidirectional WDM rings. Two forms of nonuniform traffic are specially considered in this letter due to their simplicity and reasonable depiction of reality. The evenly distributed nonuniform traffic assumes that the traffic units between two nodes are evenly distributed between 1 and the maximum limit, with the mean value \( \mu \) as specified. It provides insightful results in a statistical sense. To account for a more realistic traffic scenario, we choose the first-order linear distance-dependent traffic pattern as first suggested in [7] since, historically, nodes that are closer in distance exchange more traffic.

Figure 5(a) is the graphical output from the Matlab simulation program for uniform traffic in a bidirectional ring with 15 nodes. Squares and triangles in the diagram indicate the end nodes in the four-drop and three-drop circles, respectively. Figure 5(b) is a sample output for distance-dependent traffic in a bidirectional ring with ten nodes. Squares and triangles in the diagram denote the end nodes in four-drop and two-drop circles, respectively. Altogether 27 circles are formed by the FCCA in this case. The drops in the circles constructed by the PCCA are indicated by stars. As we can observe from Figure 5(b), all of the five circles constructed by the PCCA are full circles. In other words, the bandwidth of all of the wavelength channels is fully utilized. Actually, if we go one step further to assign up to \( g \) circles into one wavelength, and try to minimize the number of end nodes in all of the wavelengths, we would be able to obtain a design solution with the minimum number of wavelengths and an optimized number of transceivers. Further discussions on wavelength assignment and traffic grooming can be found in [4, 8].

Figure 6 shows the minimum number of wavelengths required in unidirectional [Fig. 6(a)] and bidirectional (Fig. 6(b)) rings when a connection carries a full wavelength of traffic. If one connection carries only a subwavelength of traffic, say \( 1/\alpha \) of a full wavelength, the number of required wavelengths will be exactly \( 1/\alpha \) of the corresponding value shown here. The distance-dependent traffic requires about six times the wavelengths in a unidirectional ring than what is needed in one direction on a bidirectional ring. This suggests that bidirectional rings generally would be more economical for localized traffic in terms of fiber costs since traffic is always routed over the shortest path from the source node to the destination.

\[ 5. \text{ CONCLUSION} \]

This letter has presented a series of matrix-based circle-construction algorithms to minimize the number of wavelengths in WDM ring networks for an arbitrary traffic pattern. The

For unidirectional rings,
- initialize the \( N(N - 1)/2 \) circle matrices constructed by joining the source and destination;
- \( s = N - 1; \) \hspace{1cm} // the longest connection in unidirectional rings

For bi-directional rings,
- if \( N \) is odd,
  - initialize the \( (N^2 - 1)/8 \) circle matrices constructed by CATS;
  - \( s = (N - 1)/2; \) \hspace{1cm} // the longest connection in bi-directional rings when \( N \) is odd
- if \( N \) is even, \( s = N/2; \)
  - initialize the \( N^2/8 \) circle matrices constructed by CADS;
  - \( s = N/2; \) \hspace{1cm} // the longest connection in bi-directional rings when \( N \) is even
- \( C_i = 0, C_j = 0; \) // \( C_i \) and \( C_j \) are the circles constructed using FCCA and PCCA respectively

// implementations of FCCA
for each circle matrix
  \( C_i = C_i + 1; \)
  traffic matrix \( G \) subtracted by the circle matrix;
end
// implementations of PCCA, in which longer connections are considered first
for \( i = s : -1 : 1 \)
  for \( j = 1 : N \)
    \( G(i, s) > 0 \)
    if it can be fitted into a partial circle
      update the matrix of that partial circle;
      \( G(i, s) = G(i, s) - 1; \)
    end
    if it cannot be fitted into any of the existing \( C_j \) circles
      create a new circle matrix to hold the current connection;
      \( C_j = C_j + 1; \)
      \( G(i, s) = G(i, s) - 1; \)
  end
end

Algorithm I Minimizing Wavelengths (W) for Nonuniform Traffic Using FCCA and PCCA
Matlab simulation program based on this algorithm can effectively generate the optimum network design in terms of fiber cost. Most importantly, the proposed algorithms can serve as the foundation for other network optimization algorithms to produce the desired network solutions based on a group of criteria (e.g., the total terminal equipment cost associated with electronic multiplexing), in addition to the number of wavelengths.

REFERENCES


