Mixture Models for Dynamic Statistical Pressure Snakes

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Abstract

This paper introduces a new approach to statistical pressure snakes. It uses statistical modeling for both object and background to obtain a more robust pressure model. The Expectation Maximization (EM) algorithm is used to model the data into a Mixture of Gaussians (MoG). Bayesian theory is then employed as a decision making mechanism. Experimental results using the traditional pressure model and the new mixture pressure model demonstrate the effectiveness of the new models.

1. Introduction

Active contour models, more commonly known as snakes, are a rapidly evolving area of computer vision. Since they were introduced by Kass et al. [1], active deformable contours have been widely used in several computer vision applications ranging from robotic applications [2] to medical image processing [3].

Active contour models are energy-minimizing splines that deform to fit specified image features influenced by different forces. These forces consist of internal forces, image forces, and external forces. Allowing the snake to change its shape and position minimizes the energy of the contour. The energy of the snake is given by

$$E_S = \int_0^1 (E_{Int}(S(u)) + E_{Ext}(S(u)) + E_{Img}(S(u)))du \tag{1}$$

where $S(u)$ is a parametric representation of the contour defined as

$$S(u) = (x(u), y(u))', u = [0, 1]. \tag{2}$$

In most applications of snakes, a strong edge representing an image gradient must be detected to drive the image force of the contour, which causes the same to collapse or perform poorly in weak gradient fields. Also, this type of snakes must be carefully placed so that part of the contour crosses the gradient that is associated with the object of interest. Ivins and Porrill [4] proposed a statistical approach *(statistical pressure snakes)* for computing image pressure forces, where edge energy is replaced by region energy that is a function of the statistical characteristics of the object of interest. Pressure snakes alleviated the need to detect strong edges in the image. Unfortunately, these pressure snakes have several user-defined parameters that must be hand-tuned to achieve reasonable segmentation results.

In this paper, we introduce a new statistical approach for dynamic statistical pressure contours that eliminates the required manual parameter tuning of previous models. Also, we propose a more robust pressure model that can be used in environments with complex shading and coloring. Finally, we show that our method eliminates the need to carefully place the initial snake by hand.

This paper is organized as follows. In Section 2 we introduce the automatic selection of the parameters of traditional pressure snakes. We present the new pressure model in Section 3. Section 4 gives the experimental results that demonstrate the improved performance of our method. We draw conclusions and suggest some future research directions in Section 5.

2. Automatic Selection of Pressure Snake Parameters

Ivins and Porrill [4] used first order statistics to drive their snakes. Their pressure model was given by

$$F(S) = \left(\frac{\partial S}{\partial u}\right)^{\perp}\left(1 - \frac{|I(S) - \mu|}{k\sigma}\right). \tag{3}$$

where S is the contour, μ and σ are the mean and the standard deviation of a user-defined seed region, and k is a user-defined parameter for the spread of the population. It is obvious from Equation (3) that this model assumes a single Gaussian distribution for the seed area. From Equation (3), we see that a positive pressure is applied if the distance between the image intensity and the mean is within $k\sigma$ and negative pressure is applied otherwise. Hence, the accurate selection of k is required to obtain good snake performance. It also implies that a negative pressure will be applied to the snake if the local image intensity is outside the $k\sigma$ region, even if that local intensity is vastly different from the object intensity.

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To estimate the optimal value of k, we use the following approach:

Step 1

Using a mixture of two Gaussian distributions, estimate the underlying probability density function (*pdf*) of the gray levels of the image pixels using the EM algorithm as described in [5]

Step 2

According to Bayesian decision theory, the optimal decision boundary $\mathbf{x}^*$ between the two distributions can be calculated by solving the equation

$$\frac{\pi_1}{\sqrt{2\pi\sigma_1^2}}\exp\left(-0.5\frac{(\mu_1-\mathbf{x}^*)^2}{\sigma_1^2}\right) = \frac{\pi_2}{\sqrt{2\pi\sigma_2^2}}\exp\left(-0.5\frac{(\mu_2-\mathbf{x}^*)^2}{\sigma_2^2}\right). \quad (4)$$

This yields two values for $\mathbf{x}^*$. We chose the solution that lies between the means of the two components;

Step 3

From our *a priori* knowledge about the seed region, we know which Gaussian represents the object. The value for k is then given by

$$k = \frac{|\mu_O - \mathbf{x}^*|}{\sigma_O}. \quad (5)$$

Figure 1 through Figure 4 show the result of applying the proposed algorithm.

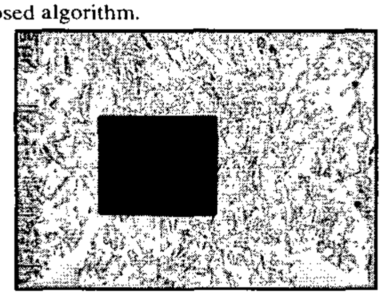

Figure 1 Sample image

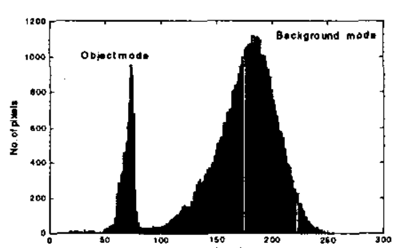

Figure 2 Histogram of Figure 1

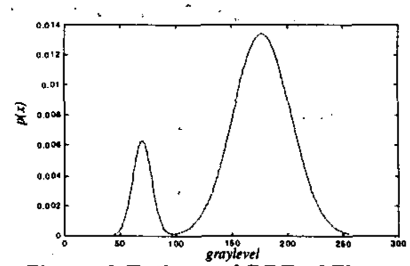

Figure 3 Estimated PDF of Figure 1

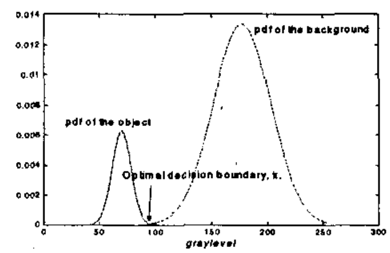

Figure 4 PDF's and optimal decision boundary

3. The New Pressure Model

The motivation to develop a more robust pressure model is that the assumption that both the object and the background are homogeneously colored and can be modeled by a single Gaussian for each is not valid in most situations. The more common case is to have a multicolored object and/or background. In the general case, a single Gaussian distribution will represent neither the object nor the background, and no decision boundary can be computed analytically. Hence, using first-order statistics will not be sufficient to represent the image data. Figure 5 is an example of a typical complex colored background. Figure 6 shows the multimodal nature of the background.

Our proposed approach consists of the following steps (where B represents the background and O represents the object):

Step 1

Estimate the underlying *pdf* of background, $p(\mathbf{x}|B)$, using the EM algorithm.

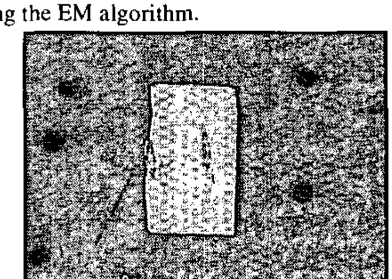

Figure 5 A complex colored background

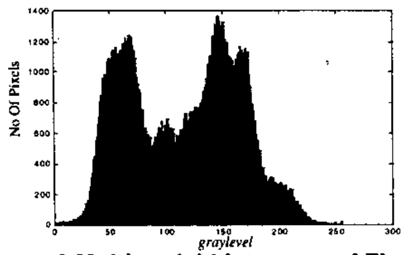

Figure 6 Multimodal histogram of Figure 5

Step 2

Estimate the *pdf* function of object, $p(\mathbf{x}|O)$.

Step 3

Using $p(\mathbf{x}|B)$ and $p(\mathbf{x}|O)$, segment out the object to obtain the initial seed region;

Step 4

Iterate the snake using the pressure model given by

$$F(S) = (p(\mathbf{x}|O) - p(\mathbf{x}|B))\left(\frac{\partial S}{\partial u}\right)^{\perp} \qquad (6)$$

The term $(p(\mathbf{x}|O) - p(\mathbf{x}|B))$ represents the magnitude and the direction of the pressure, while the $(\partial S)/(\partial u)$ term represents the split of the pressure in. x and y direction. The number of components in each mixture can be automatically computed using any technique such as [7].

Figure 7 through Figure 10 illustrate the results of each step of the proposed approach.

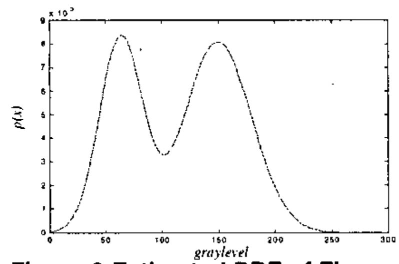

Figure 9 Estimated PDF of Figure 5

Figure 7 Simple colored object

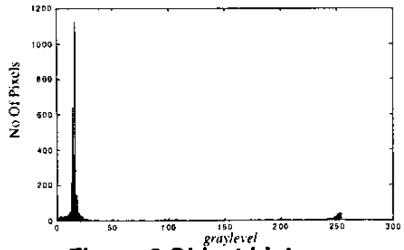

Figure 8 Object histogram

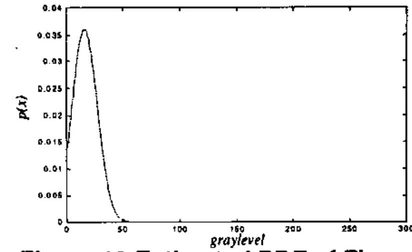

Figure 10 Estimated PDF of Figure 7

4. Experimental Results

Figure 11 shows the result of applying the traditional pressure snake formulation on a simple colored object. We can see that the snake cannot fit to the object edges because of the imbalanced object and background pressures.This phenomenon of "hovering" inside the actual object contour is typical of dynamic statistical pressure snakes. Figure 12 shows the results using the new pressure model. We can see that the snake conforms properly to the object edges because of the balance between the object's pressure and the background's pressure due to using their *pdf*'s. This is a significant performance improvement over the original models.

Figure 13 shows the estimated *pdfs* of a complex colored object (checker board) and a complex colored background (another checker board). Figure 14 shows the result of applying the new pressure model to the segment the object. When applying the original snake formulation to

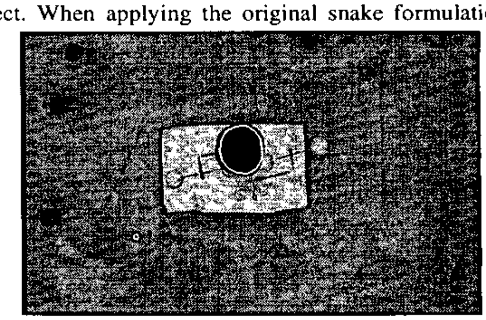

Figure 11 Performance of the original pressure

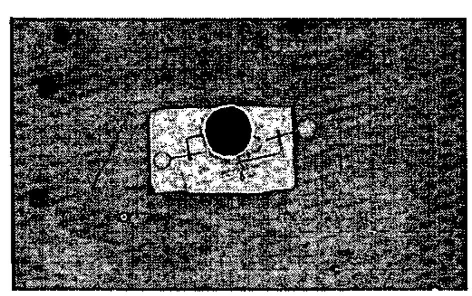

Figure 12 Performance of the mixture pressure

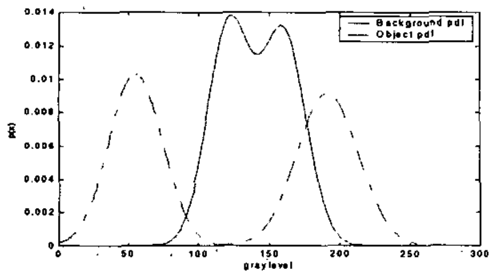

Figure 13 PDFs of a complex colored object and background

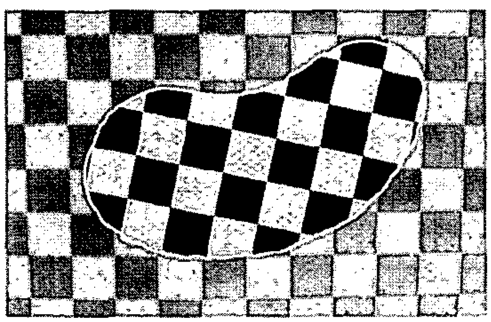

Figure 14 Performance of new pressure model

this object, the snake collapsed because of the overlap between the color distributions of the object and that of the background. This means that there is no sharp decision boundary between the object and the background which causes the original formulation to fail.

We used the new formulation in an object tracking application. We achieved a 30 frames/sec. tracking rate, plus a period of about 1 second added to the startup of the application. The increased startup time is for the estimation of the *pdfs* and initializing the snake. Another approach that we used was to initialize the snake to a rectangle slightly smaller than the image size. The snake collapsed to the edges of the object due to the effects of the background pressure. In practice this worked very well.

This method of initializing the snake eliminates the stringent placement required by earlier snakes, which was one of the crucial shortcomings of classic active contours.

5. Conclusions and Future Research

In this paper, we presented a new statistical approach for pressure snakes using mixture models. This new approach eliminates the hand-tuning of pressure model parameters and the precise initial placement of the snakes.

Two separate cases have been treated. The first case is when the object data and the background data can be represented by a single Gaussian for each. For this case, we introduced a new method for calculating the parameters of the classic pressure model (μ, σ and k.) by computing the optimal decision boundary according to Bayes theory.

The second case is the general case, where neither the object nor the background (or both) is complex colored, and cannot be represented by a single Gaussian. In this case, we used the EM algorithm to fit the data to a mixtures of Gaussian distributions. Then we use a new formulation for the pressure model based upon the *pdf*'s to iterate the active contour.

Another improvement was the ability to place the snake just inside the image boundaries. This improvement alleviated the problem of manually placing the classic snakes. Also, using our new formulation, it becomes easy to place several snakes on the image each looking for a different object with different statistical properties (i.e. *pdf*.)

The performance of the proposed algorithm was evaluated against the performance of the classic active pressure contours using an object tracking application as well as other complex colored objects. The experiments demonstrated a significantly improved object segmentation while maintaining the real time performance of the models.

It is woth mentioning here that, in case of very similar object and background *pdfs*, not only our algorithm will fail, but also all algorithms that are based on Bayes theory and the gray level as a feature. In this case, our method will still work if we model another feature that diffrentiates the object from the background (e.g. texture.)

For future research, we want to adapt this technique to other forms of pressure models, including color, spatial frequency, and texture.

6. References

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