1 Problem (Homework 7)

Using the multi-level fast multipole method, compute the matrix-vector product

$$\mathbf{v} = \Phi \mathbf{u}, \tag{1}$$

or

$$v_j = \sum_{i=1}^{N} \Phi_{ji} u_i, \quad j = 1, ..., M, \tag{2}$$

with absolute error $\epsilon < 10^{-6}$,where

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} & ... & \Phi_{1N} \\ \Phi_{21} & \Phi_{22} & ... & \Phi_{2N} \\ ... & ... & ... & ... \\ \Phi_{M1} & \Phi_{M2} & ... & \Phi_{MN} \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ ... \\ u_N \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ ... \\ v_M \end{pmatrix}, \tag{3}$$

$$\Phi_{ji} = \frac{1}{y_j - x_i}, \quad i = 1, ..., N, \quad j = 1, ..., M.$$

and $x_1, ..., x_N$ are random points uniformly distributed on [0,10], $M = N - 1$, and each y_j is located between the closest x_i's on each side, $j = 1, ..., N - 1$.

Homework 7

1. Derive expressions for S|S and R|R operators and implement them. Check if your program works correctly.
2. Write a program that implements both straightforward multiplication based on Eq. (2) and Multi Level FMM (MLFMM).
3. Provide a graph of the absolute maximum error between the straightforward and the MLFMM method for $N = 10^3$, and grouping parameter s varying between 1 and 100, and several p ($p \sim 10$).
4. Provide a graph of the CPU time vs s at fixed p that insures that the required accuracy is achieved. Find the optimum s for your implementation.
5. Provide a graph that compares the CPU time required by the straightforward method and the MLFMM for N varying between 10^2 and 10^3 for straightforward and N varying between 10^2 and 10^4 for the MLFMM (use the optimum s found).
6. Find the "break-even" point (i.e. N at which the "Fast" method requires the same CPU time as the straightforward method) for your implementation.
7. Provide a graph of actual error (between the standard and the fast methods) for N varying between 10^2 and 10^3 and the truncation numbers used.

Hints

Use your previous homework programs to set the data structure, compute S|R-translation operators, and straightforward solution. Separate the part that sets data structure from the run part of the MLFMM. For comparisons measure only the CPU time required for the run part of the MLFMM. You can also compare the efficiency of your program that sets data structure by comparing the CPU time for the two parts of the MLFMM.