

CMSC698R/AMSC878R Final Examination

1. (40 points) Below are some very short questions (mainly definitions to check if you have read the notes). Be concise in answering these questions. Where necessary illustrate with a rough sketch.

- (a) (3 points) What is the main reason to use the FMM for matrix-vector multiplication?
- (b) (5 points) How can the FMM be used to solve a system of linear equations?
- (c) (6 points) What is the S|R translation operator? Draw a sketch showing the domains of expansion for S|R translation.
- (d) (3 points) What is the Legendre polynomial?
- (e) (3 points) What are the spherical harmonics?
- (f) (6 points) Let $\Phi(y, x)$ be a function. What are the requirements on Φ so that the sums

$$v(y_j) = \sum_{i=1}^N \Phi(y_j, x_i) u_i, \quad j = 1, \dots, M$$

can be evaluated using the FMM?

- (g) (5 points) What is the reason to use rotation-coaxial translation decomposition of general translation operators (e.g. for the 3D Laplace equation)?
 - (h) (5 points) What other than the 3-neighborhoods could be considered for use in the MLFMM? Why?
 - (i) (2 points) What is the asymptotic complexity of the adaptive MLFMM?
 - (j) (2 points) What is the asymptotic complexity of the step that sets the data structure for MLFMM?
2. (30 points) A simpler version of the FMM algorithm than was derived in class is considered in this problem. We have a function $\Phi(y, x_i)$ which is singular at $y = x_i$. We wish to evaluate the matrix vector product

$$v(y_j) = \sum_{i=1}^N \Phi(y_j, x_i) u_i, \quad j = 1, \dots, M$$

The points $\{y_j\}$ and $\{x_i\}$ are distributed uniformly in the domain (but no points in the set $\{y_j\}$ are coincident with $\{x_i\}$, so that the sum is defined). The domain is uniformly divided into K boxes. The strategy taken is to form R -expansions of the functions $\Phi(y, x_i)$ about the box center containing evaluation points, but whose neighborhood does not contain x_i , and consolidate the expansions. The contribution of sources from the neighborhood of the evaluation box is computed directly. Determine the optimal number of boxes K as a function of N , and the overall complexity of the algorithm.

3. (30 points) Find all neighbors for box #65 at level 4 in the quad-tree where the hierarchical indexing with bit-interleaved coordinates is used.