

CMSC698R/AMSC878R Mid Term Examination

1. (30 points) Below are some very short questions (mainly definitions to check if you have read the notes). Be brief and concise in answering these questions. Where necessary illustrate with a rough sketch.

- (a) (2 points) Show that the number of operations needed to perform a regular product of a dense matrix with a vector requires $O(N^2)$ operations.
- (b) (5 points) Give examples of functions which have asymptotic behavior of $O(N)$, $o(N)$, and $\Theta(N)$.
- (c) (3 points) What is a local expansion? What is a far-field expansion? What kind of an expansion is a Taylor series?
- (d) (2 points) What is the Kronecker product of the matrices $C = A \otimes B$

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 6 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

- (e) (4 points) Explain how the “Compression” operator results in a reduction in the complexity of multidimensional summation.
- (f) (5 points) Let $\Phi(y, x)$ be a function. What are the requirements on Φ so that the sums

$$v(y_j) = \sum_{i=1}^N \Phi(y_j, x_i) u_i, \quad j = 1, \dots, M$$

can be evaluated using the FMM?

- (g) (5 points) What is bit-interleaving and how is it applied to create spatial ordering for a collection of points?
- (h) (4 points) Why do we need the domain E_4 in the multilevel fast-multipole algorithm.
2. (10 points): How many terms of the Taylor series do you need to approximate the function $\sin(x)$ in the range $x \in [-1, 1]$ with absolute error 10^{-8} ? (Hint: To those who have forgotten basic math. The derivative of $\sin(x)$ is $\cos(x)$, and the derivative of $\cos(x)$ is $-\sin(x)$. $\sin(0) = 0$ and $\cos(0)=1$).

3. (30 points) A simpler version of the MLFMM algorithm that was derived in class is considered in this problem.

In this example we have a function $\Phi(y, x_i)$ which is singular at $y = x_i$. We wish to evaluate the matrix vector product

$$v(y_j) = \sum_{i=1}^N \Phi(y_j, x_i) u_i, \quad j = 1, \dots, M$$

The points $\{y_j\}$ and $\{x_i\}$ are distributed uniformly in the domain (but do not overlap). The domain is uniformly divided into K boxes. The strategy to be taken is to form S expansions of the functions $\Phi(y_j, x_i)$ that lie within a box about that box center, and consolidate the expansions. These K sets of expansions are valid everywhere except for the box itself and its neighboring boxes. The consolidated S expansions at the centers of the boxes are evaluated at the evaluation points $\{y_j\}$ for which the expansion is valid, and for the remaining points, direct evaluation is employed. Determine the optimal number of boxes K as a function of N , and the overall complexity of the algorithm.

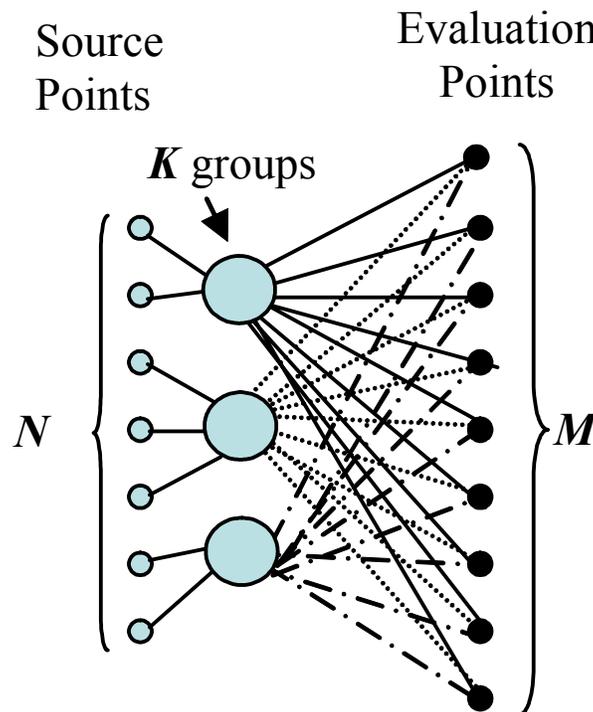


Figure 1:

- (30 points) Find all neighbors for box #127 at level 4 in the quad-tree where the hierarchical numbering (indexing) with bit-interleaved coordinates is used. (Hint: See Lecture Notes).