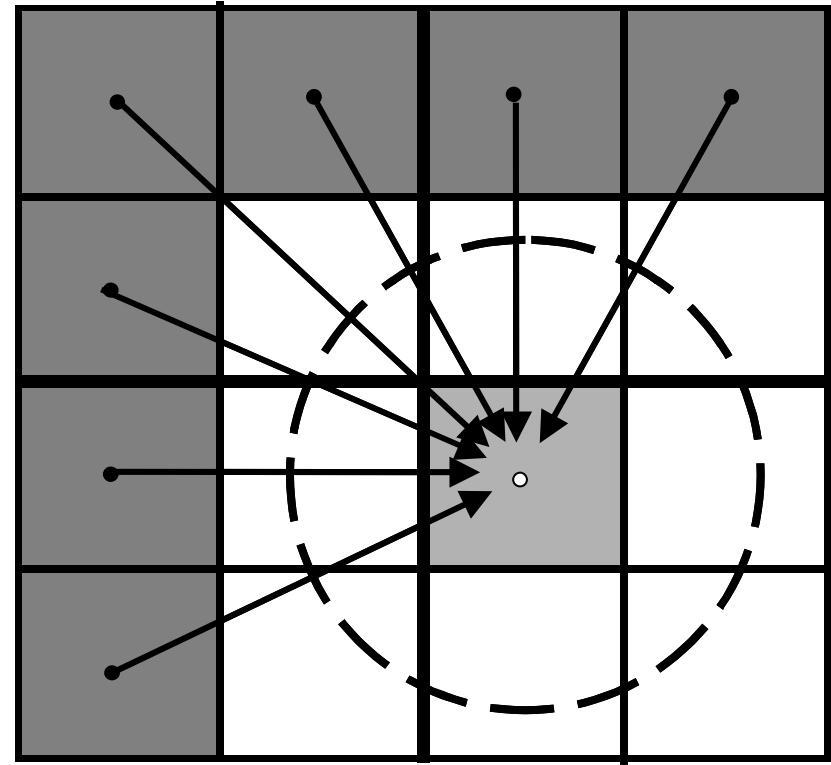
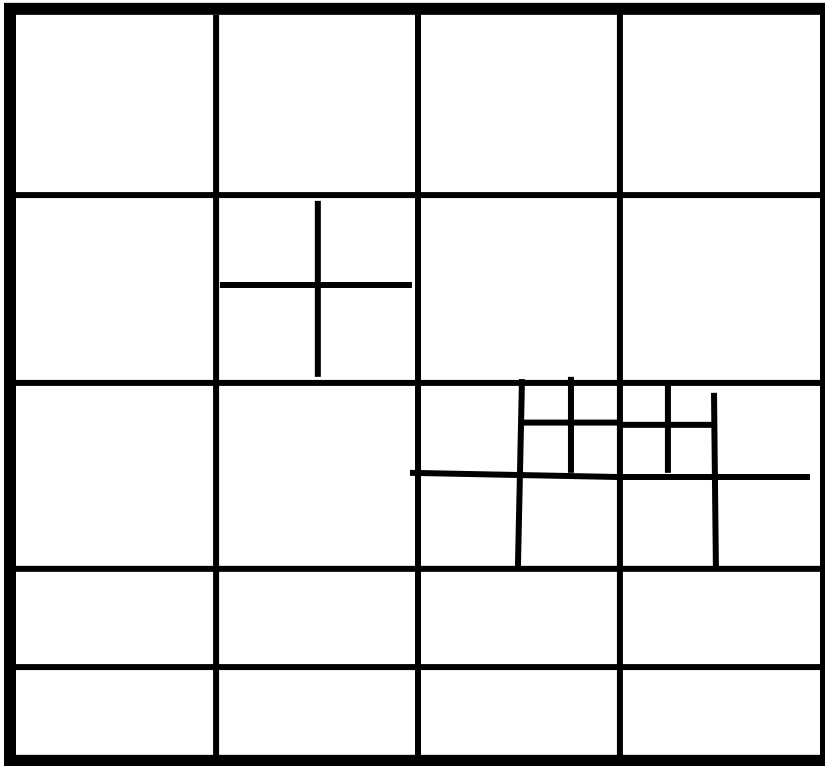
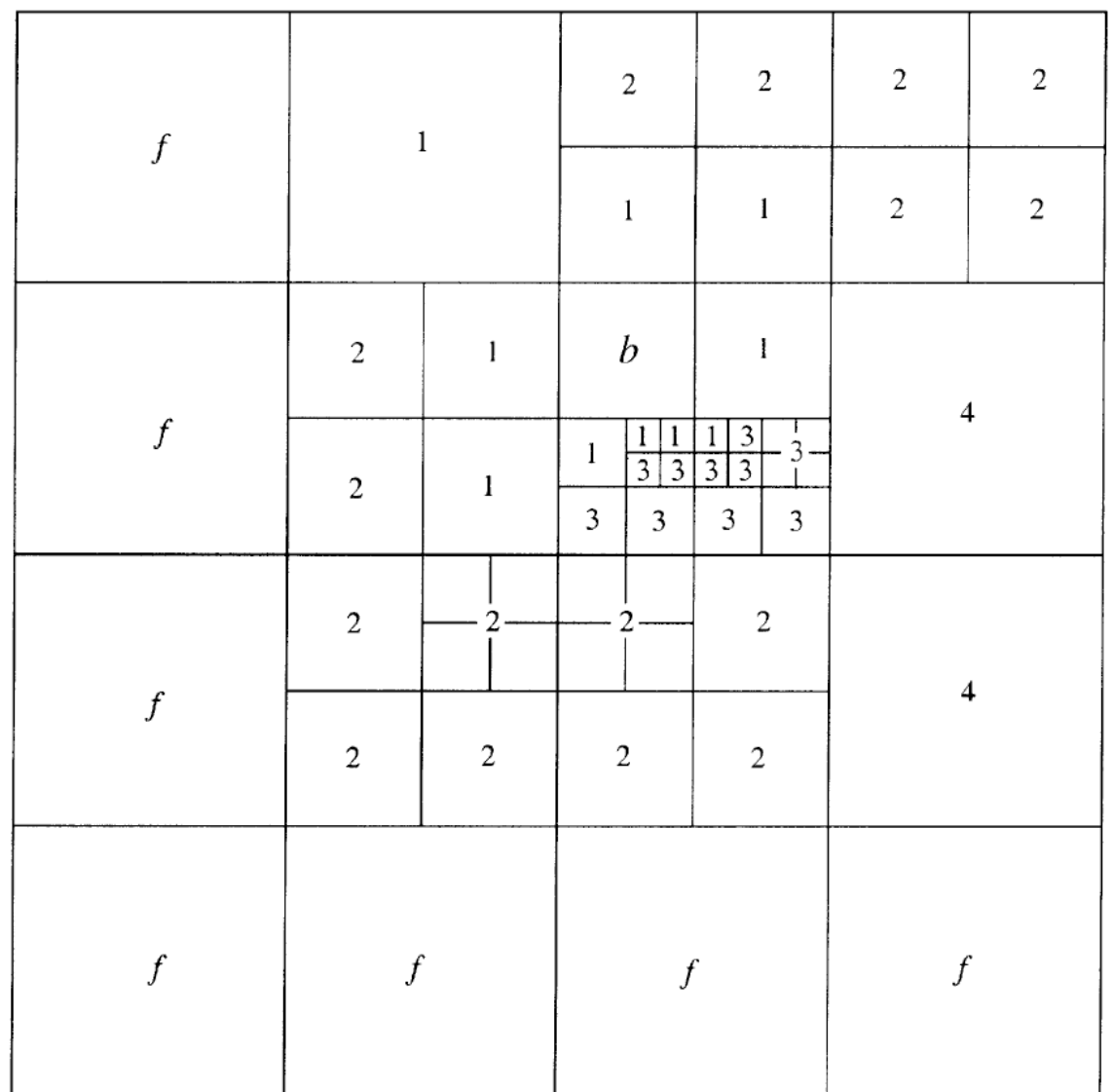


# Adaptivity

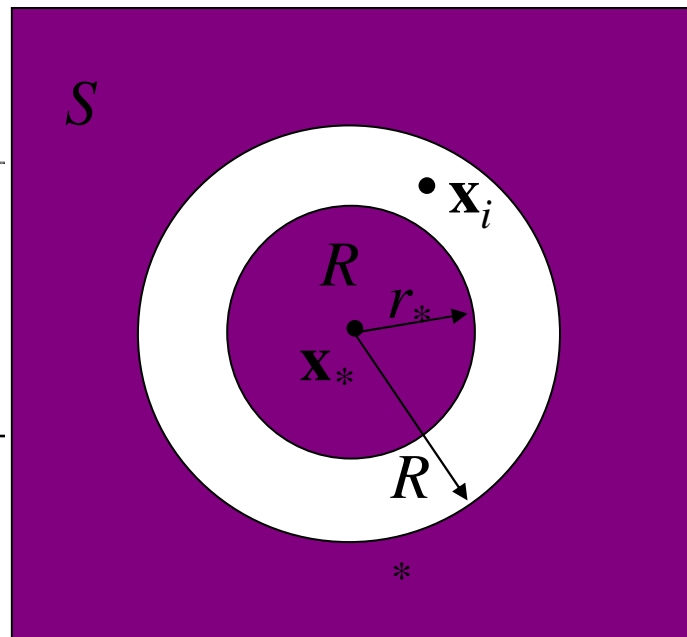


- Parameter  $s$  is determined by the box with most particles.
  - Most boxes will be empty (ok)
  - or have much fewer than  $s$  particles
- Reducing number of levels is good
- Alternate strategy
  - Continue subdivision so that each parent box has  $> \ell$  points and each leaf-box has  $\leq \ell$  points
- Consider an algorithm due to Cheng et al (J. Comput. Phys. 1999).

- At the finest level consider evaluation at box  $b$ .
- It is surrounded by boxes
  - Coarser, Same or Finer level
  - Already considered, in a coarser S|R translation
- Divide surrounding boxes not already considered in to  $L_1, L_2, L_3, L_4$  domains
  - Boxes  $L_1$  share a boundary with box  $b$ , or they lie within the sphere of box  $b$  and need to be evaluated directly.
  - Boxes  $L_2$  are separated by at least one box of size of  $b$  S|R translate these
  - Boxes in  $L_3$  contain sources that lie outside the sphere of  $b$  and are closer than boxes  $L_2$  R-preFMM these
  - Box  $b$  lies outside the sphere of boxes in  $L_4$ . but is too close to S|R. S-preFMM these



	1	1		
1	$b$	1		
1	1	1	1	3
	3	3	3	3
2	2			2



For  $x_i$  in  $L_3$  we can build R expansions in  $b$

$$r^* < R^*$$

S-expansion:  $|y - x_*| > R_* > |x_i - x_*|$

R-expansion:  $|y - x_*| < r_* < |x_i - x_*|$

# Algorithm parameters & optimization

- Suggest that  $P$  and  $I$  should be balanced but do not provide an optimal choice
- A good project
  - to implement this and find out optimal choices.
  - compare this scheme with our scheme and check efficiency
  - Perhaps develop a combined scheme.