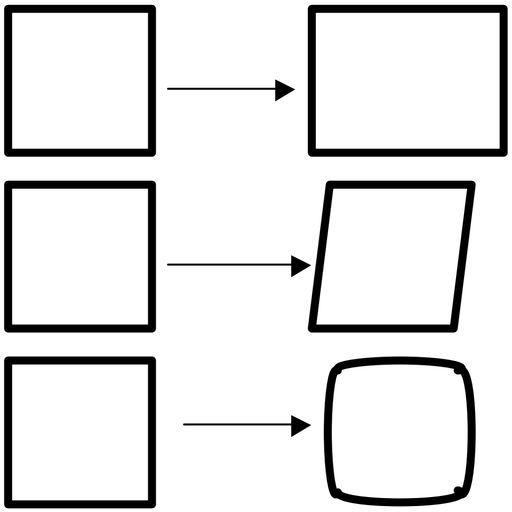
Camera Calibration

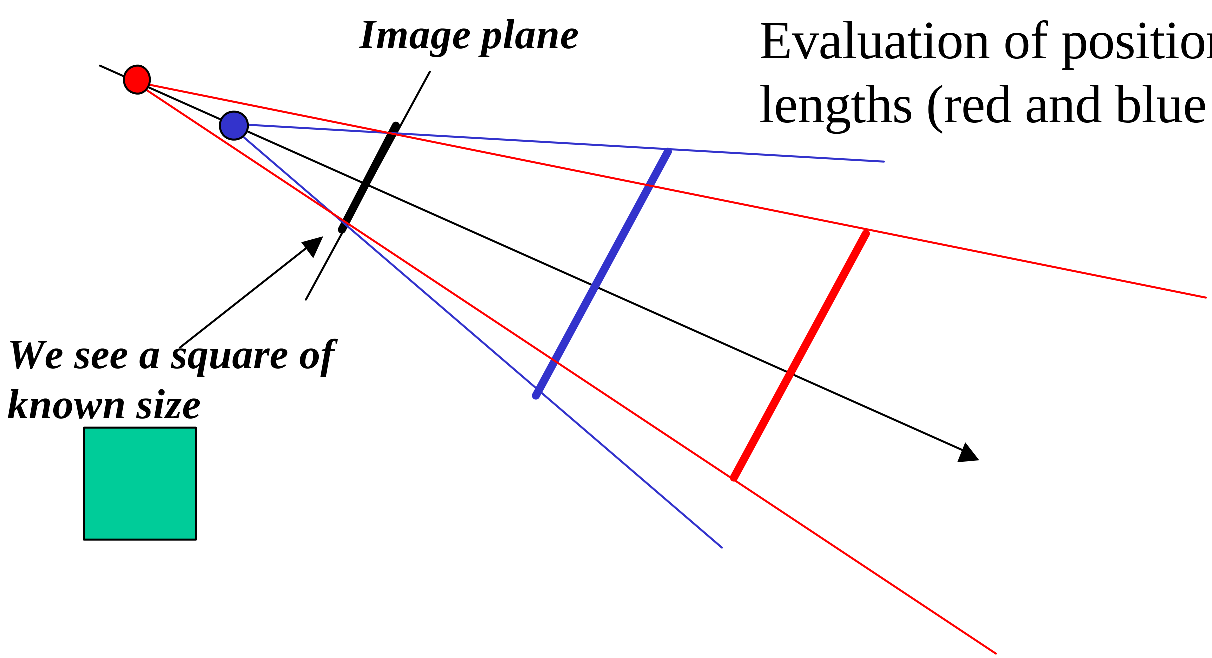
What is Camera Calibration?

- Primarily, finding the quantities internal to the camera that affect the imaging process
  - Position of image center in the image
    - It is typically not at (width/2, height/2) of image
  - Focal length
  - Different scaling factors for row pixels and column pixels
  - Skew factor
  - Lens distortion (pin-cushion effect)



Motivation

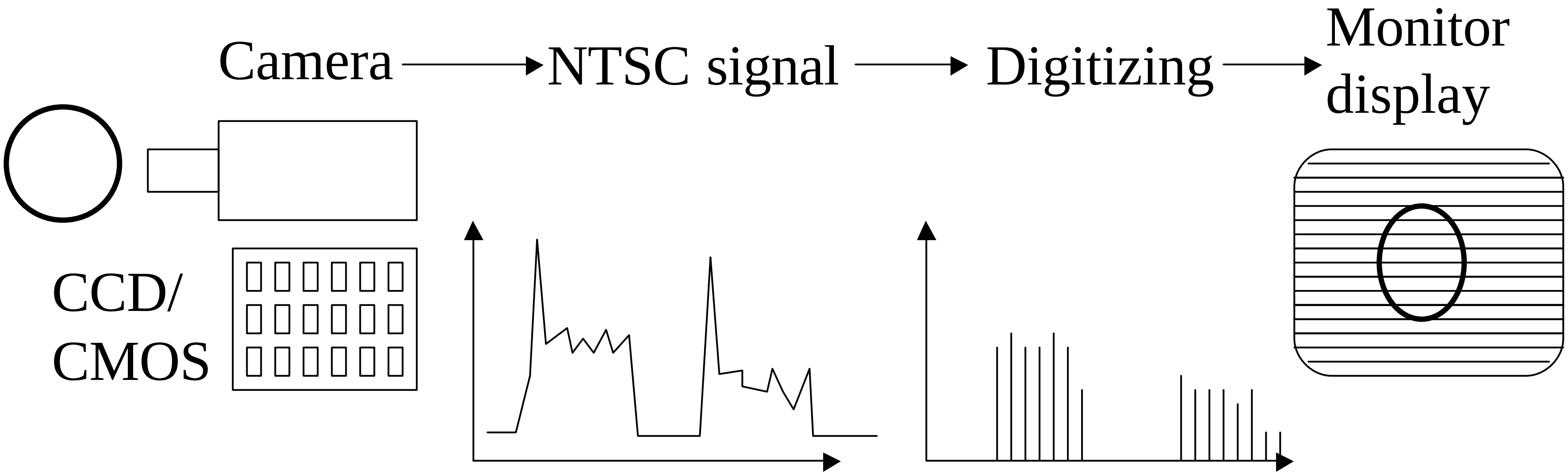
- Good calibration is important when we need to
  - Reconstruct a world model: Virtual L.A. project
  - Interact with the world
    - Robot, hand-eye coordination



Evaluation of position of a square for 2 focal lengths (red and blue projection geometry)

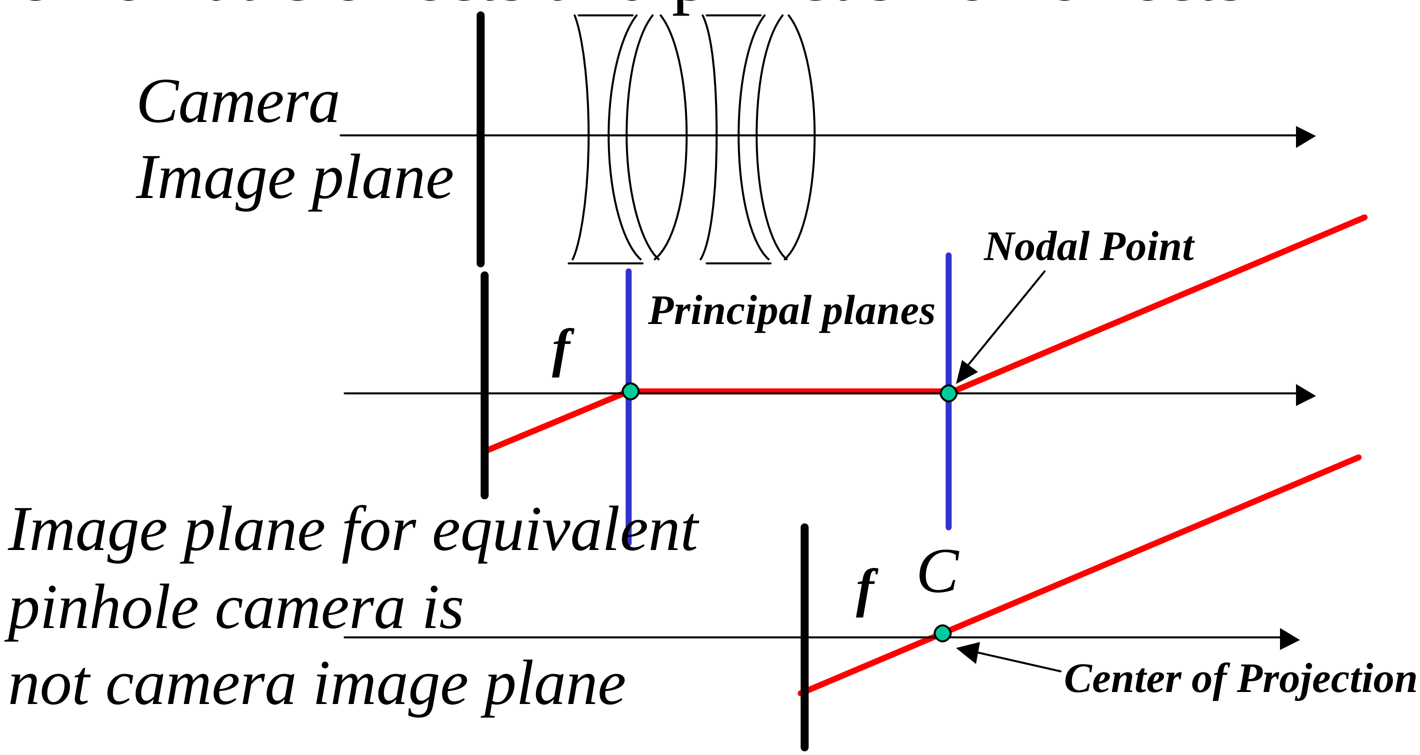
Scaling of Rows and Columns in Image

- Camera pixels are not necessarily square
- Camera output may be analog (NTSC)
- Image may be obtained by digitizing card
  - A/D converter samples NTSC signal



Compound Lens Imaging

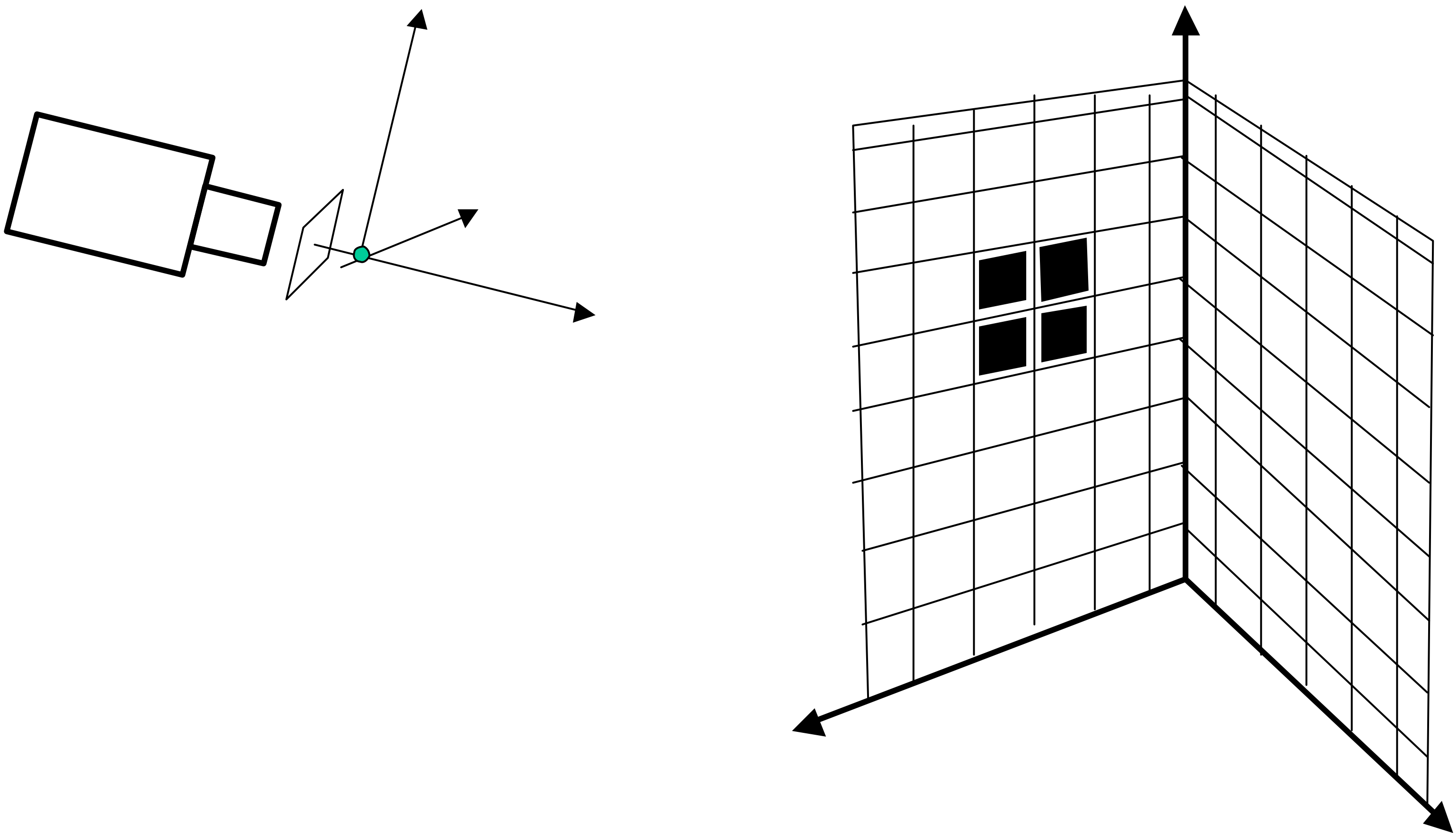
- Inexpensive single lens systems distort image at its periphery
- Compound lenses may be used to reduce chromatic effects and pin-cushion effects



Variety of Techniques

- VERY large literature on the subject
- Work of Roger Tsai influential
- Linear algebra method described here
  - Can be used as initialization for iterative non linear methods.
- Some interesting methods use vanishing points

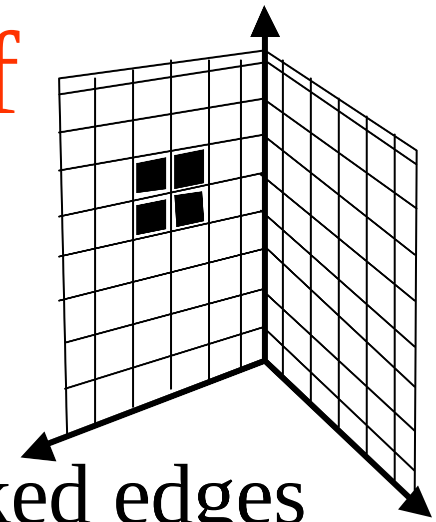
Camera and Calibration Target



Calibration Procedure

- Calibration target: 2 planes at right angle with checkerboard patterns (Tsai grid)
  - We know positions of pattern corners only with respect to a coordinate system of the target
  - We position camera in front of target and find images of corners
  - We obtain equations that describe imaging and contain internal parameters of camera
    - As a side benefit, we find position and orientation of camera with respect to target (camera ***pose***)

Image Processing of Image of Target



- Canny edge detection
- Straight line fitting to detected linked edges
- Intersecting the lines to obtain the image corners
- Matching image corners and 3D target checkerboard corners
  - By counting if whole target is visible in image
- We get pairs (image point)--(world point)

$$(x_i, y_i) \rightarrow (X_i, Y_i, Z_i)$$

Central Projection

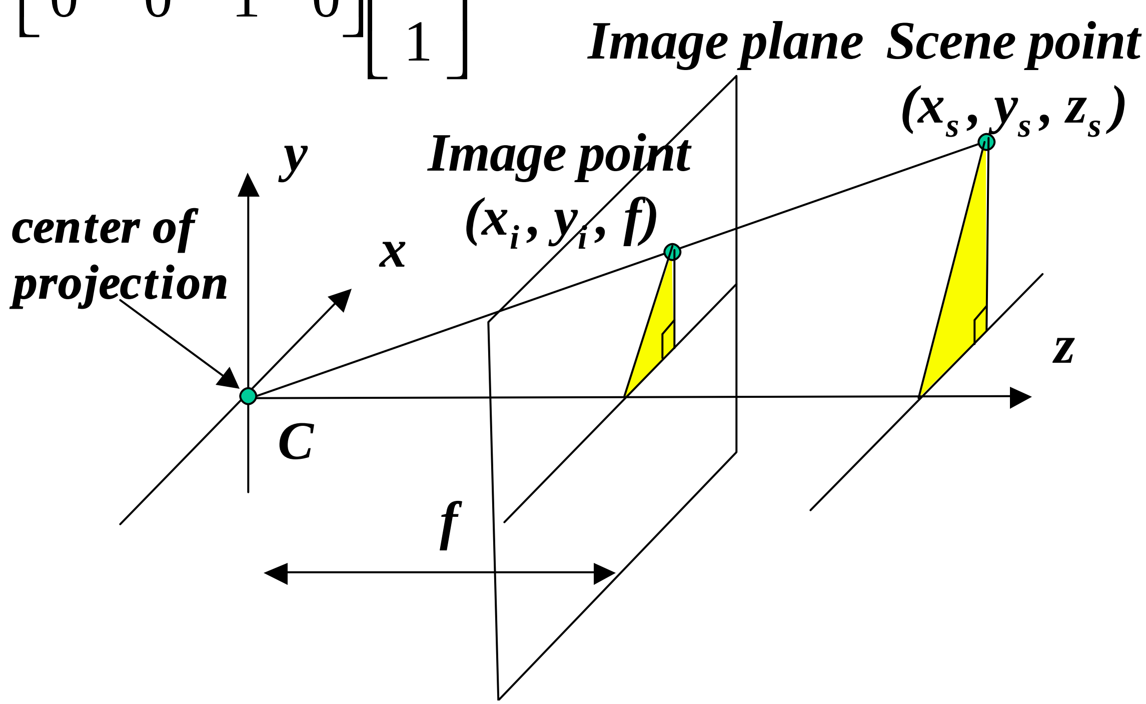
If world and image points are represented by homogeneous vectors, central projection is a linear transformation:

$$x_i = f\,\frac{x_s}{z_s}$$

$$y_i = f\,\frac{y_s}{z_s}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix}$$

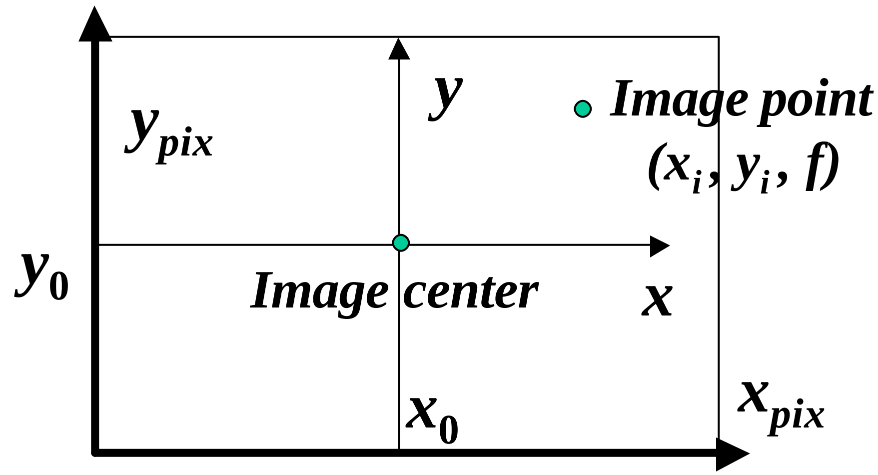
$$x_i = u / w, \quad y_i = v / w$$



Transformation From Lengths to Pixels

Transformation uses:

- image center (x_0, y_0)
- scaling factors k_x and k_y



$$x_i = f\frac{x_s}{z_s} \qquad x_{pix} = k_x x_i + x_0 = f\,k_x \frac{x_s + z_s x_0}{z_s}$$

$$y_i = f\frac{y_s}{z_s} \qquad y_{pix} = k_y y_i + y_0 = f\,k_y \frac{y_s + z_s y_0}{z_s}$$

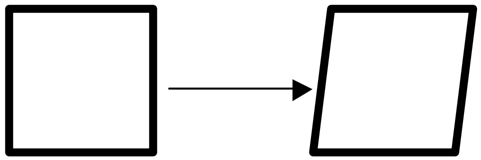
$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} a_x & 0 & x_0 & 0 \\ 0 & a_y & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix} \quad \text{with} \quad \begin{aligned} a_x &= f\,k_x \\ a_y &= f\,k_y \end{aligned} \quad \text{then} \quad \begin{aligned} x_{pix} &= u'/w' \\ y_{pix} &= v'/w' \end{aligned}$$

Internal Camera Parameters

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} a_x & s & x_0 & 0 \\ 0 & a_y & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix} \quad \text{with} \quad \begin{aligned} a_x &= f\,k_x \\ a_y &= -f\,k_y \end{aligned} \qquad \begin{aligned} x_{pix} &= u'/w' \\ y_{pix} &= v'/w' \end{aligned}$$

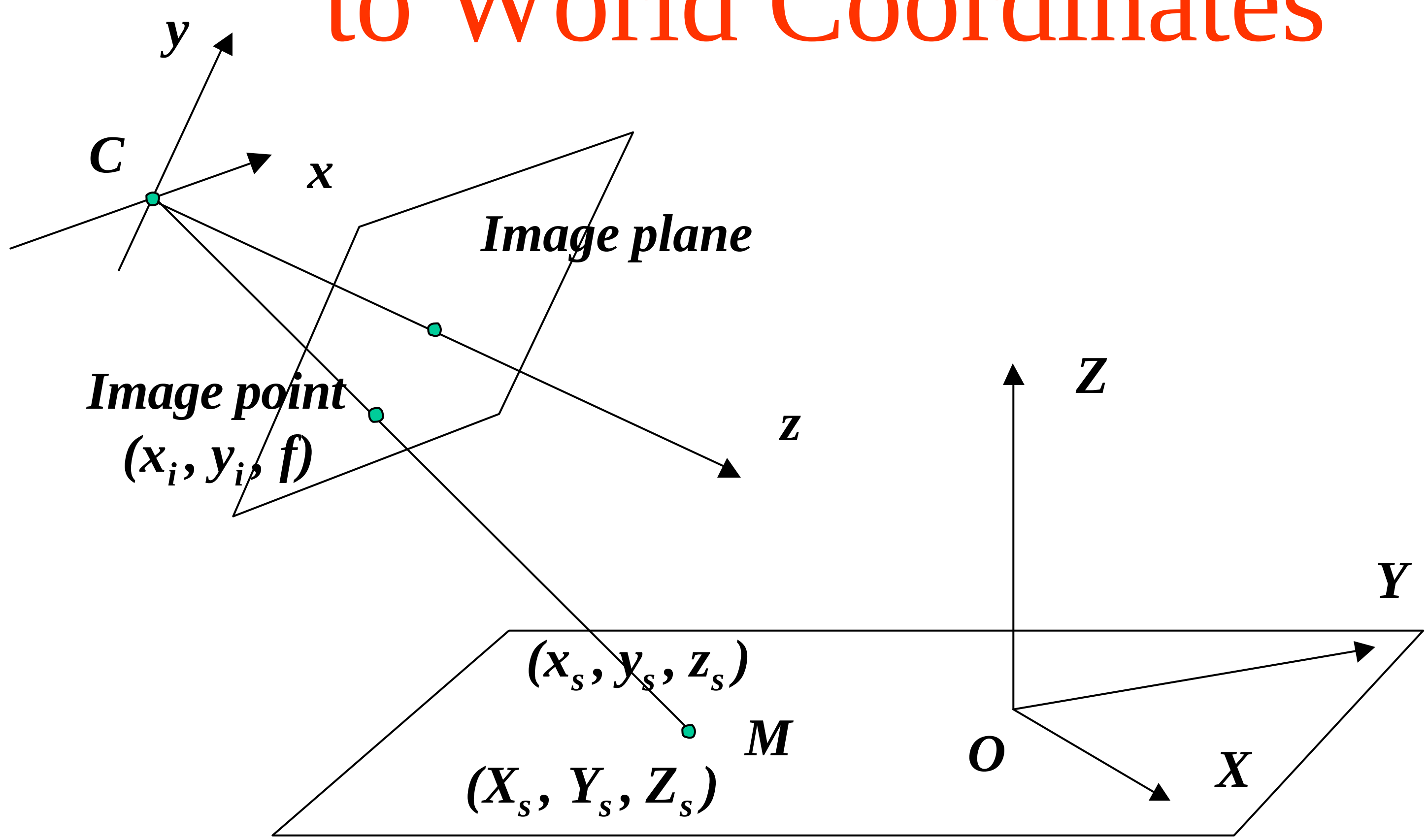
$$\begin{bmatrix} a_x & s & x_0 & 0 \\ 0 & a_y & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} a_x & s & x_0 \\ 0 & a_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \mathbf{K}\left[\mathbf{I}_3 \quad | \quad \mathbf{0}_3\right]$$

- a_x and a_y “focal lengths” in pixels
- x_0 and y_0 coordinates of image center in pixels
- Added parameter S is skew parameter

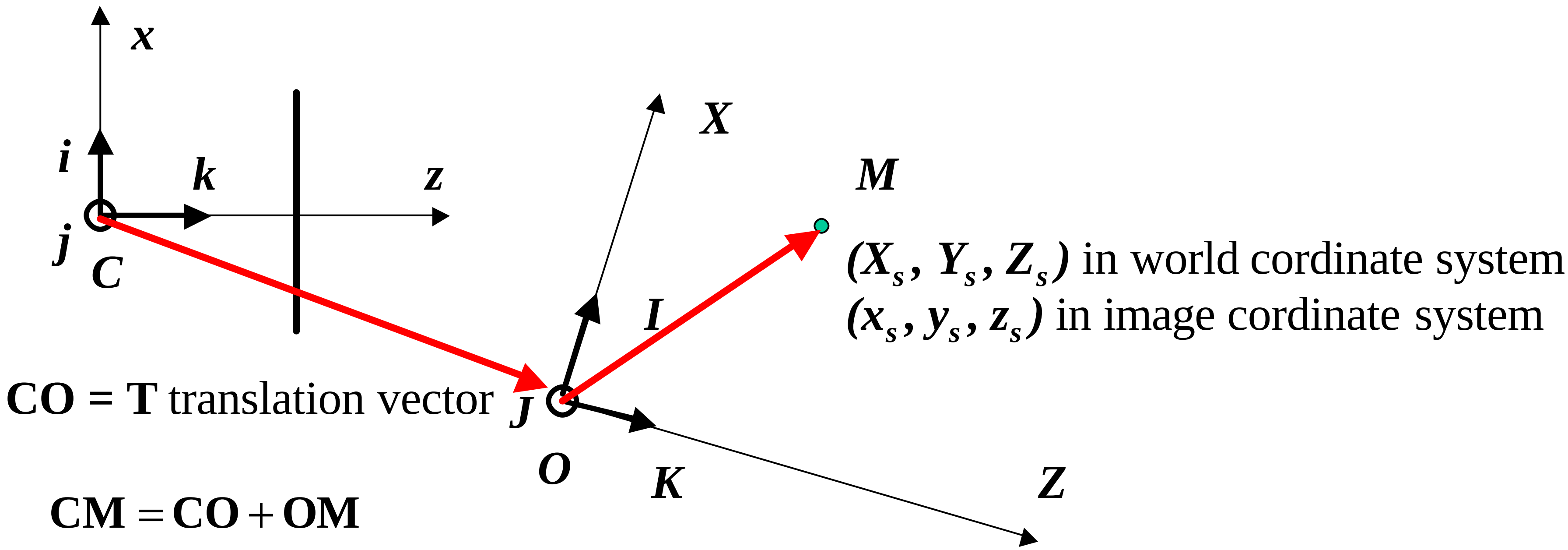


- **K** is called *calibration matrix*. **Five degrees of freedom.**
  - **K** is a 3x3 upper triangular matrix

From Camera Coordinates to World Coordinates



From Camera Coordinates to World Coordinates 2



$$\mathbf{CM} = \mathbf{CO} + \mathbf{OM}$$

$$x_S\mathbf{i} + y_S\mathbf{j} + z_S\mathbf{k} = T_x\mathbf{i} + T_y\mathbf{j} + T_z\mathbf{k} + X_S\mathbf{I} + Y_S\mathbf{J} + Z_S\mathbf{K}$$

$$x_S = T_x + X_S\mathbf{I.i} + Y_S\mathbf{J.i} + Z_S\mathbf{K.i}$$

$$\begin{bmatrix} x_S \\ y_S \\ z_S \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \begin{bmatrix} \mathbf{I.i} & \mathbf{J.i} & \mathbf{K.i} \\ \mathbf{I.j} & \mathbf{J.j} & \mathbf{K.j} \\ \mathbf{I.k} & \mathbf{J.k} & \mathbf{K.k} \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}$$

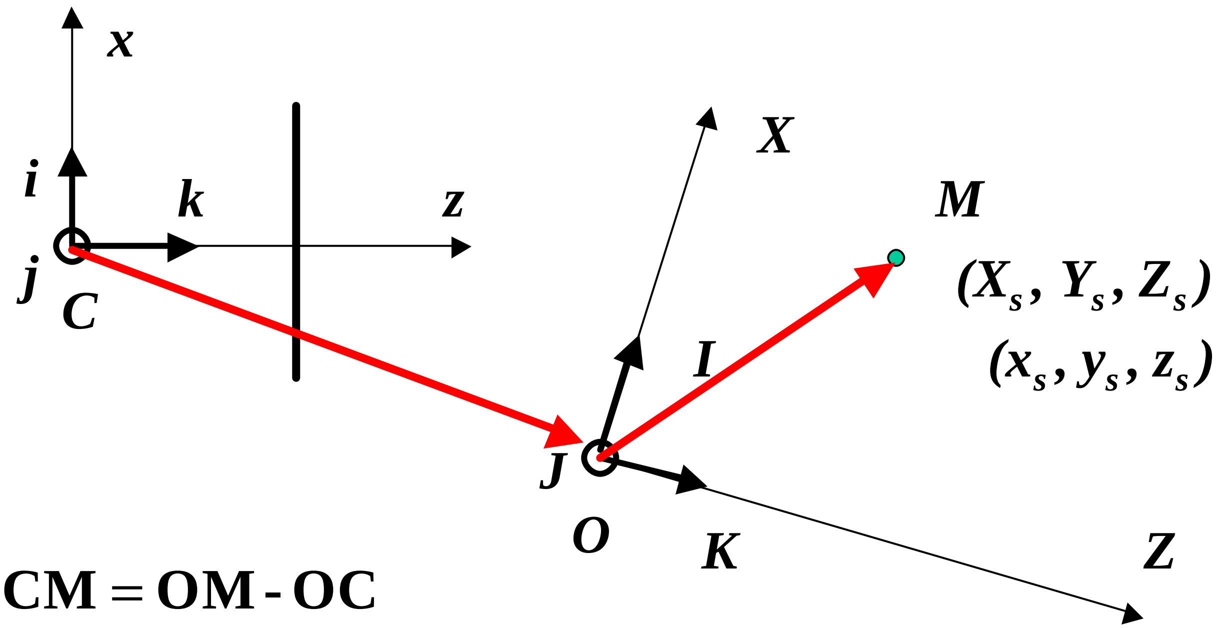
Homogeneous Coordinates

$$\begin{bmatrix} x_S \\ y_S \\ z_S \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \begin{bmatrix} \mathbf{I.i} & \mathbf{J.i} & \mathbf{K.i} \\ \mathbf{I.j} & \mathbf{J.j} & \mathbf{K.j} \\ \mathbf{I.k} & \mathbf{J.k} & \mathbf{K.k} \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}$$

$$\begin{bmatrix} x_S \\ y_S \\ z_S \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{I.i} & \mathbf{J.i} & \mathbf{K.i} & T_x \\ \mathbf{I.j} & \mathbf{J.j} & \mathbf{K.j} & T_y \\ \mathbf{I.k} & \mathbf{J.k} & \mathbf{K.k} & T_z \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_S \\ y_S \\ z_S \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{T} \\ \mathbf{0}_3^\mathbf{T} & 1 \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \\ 1 \end{bmatrix}$$

From Camera Coordinates to World Coordinates 3



$$\mathbf{CM} = \mathbf{OM} - \mathbf{OC}$$

$$x_S \mathbf{i} + y_S \mathbf{j} + z_S \mathbf{k} = (X_S - X_C)\mathbf{I} + (Y_S - Y_C)\mathbf{J} + (Z_S - Z_C)\mathbf{K}$$

$$x_S = (X_S - X_C)\mathbf{I.i} + (Y_S - Y_C)\mathbf{J.i} + (Z_S - Z_C)\mathbf{K.i}$$

$$\mathbf{x}_{\mathbf{cam}} = \mathbf{R}(\mathbf{X} - \tilde{\mathbf{C}}) \qquad (\mathbf{T} = -\mathbf{R}\tilde{\mathbf{C}})$$

$\tilde{\mathrm{C}}$ is vector OC expressed in world coordinate system

Homogeneous Coordinates 2

- Here we use $-\mathbf{R}\tilde{\mathbf{C}}$ instead of $\mathbf{T}$

$$\begin{bmatrix} x_S \\ y_S \\ z_S \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0}_3^{\mathbf{T}} & 1 \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \\ 1 \end{bmatrix}$$

Linear Transformation from World Coordinates to Pixels

- Combine camera projection and coordinate transformation matrices into a single matrix **P**

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \mathbf{K}\left[\mathbf{I}_3 \quad | \quad \mathbf{0}_3\right] \begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_S \\ y_S \\ z_S \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\,\tilde{\mathbf{C}} \\ \mathbf{0}_3^{\mathbf{T}} & 1 \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \mathbf{K}\left[\mathbf{I}_3 \quad | \quad \mathbf{0}_3\right] \begin{bmatrix} \mathbf{R} & -\mathbf{R}\,\tilde{\mathbf{C}} \\ \mathbf{0}_3^{\mathbf{T}} & 1 \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \mathbf{P} \begin{bmatrix} X_S \\ Y_S \\ Z_S \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{P}\,\mathbf{X}$$

Properties of Matrix **P**

- Further simplification of **P**:

$$\mathbf{x} = \mathbf{K}\left[\mathbf{I}_3 \quad | \quad \mathbf{0}_3\right]\begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0}_3^T & 1 \end{bmatrix}\mathbf{X}$$

$$\left[\mathbf{I}_3 \quad | \quad \mathbf{0}_3\right]\begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0}_3^T & 1 \end{bmatrix} = \left[\mathbf{R} \quad -\mathbf{R}\tilde{\mathbf{C}}\right] = \mathbf{R}\left[\mathbf{I}_3 \quad | \quad -\tilde{\mathbf{C}}\right]$$

$$\mathbf{x} = \mathbf{K}\,\mathbf{R}\left[\mathbf{I}_3 \quad | \quad -\tilde{\mathbf{C}}\right]\mathbf{X}$$

$$\boxed{\mathbf{P} = \mathbf{K}\,\mathbf{R}\left[\mathbf{I}_3 \quad | \quad -\tilde{\mathbf{C}}\right]}$$

- **P** has 11 degrees of freedom:
  - 5 from triangular calibration matrix **K**, 3 from **R** and 3 from $\tilde{\mathbf{C}}$
- **P** is a fairly general 3 x 4 matrix
  - left 3x3 submatrix **KR** is non-singular

Calibration

- 1. Estimate matrix **P** using scene points and their images
- 2. Estimate the interior parameters and the exterior parameters

$$\mathbf{P} = \mathbf{K}\,\mathbf{R}\left[\mathbf{I}_3 \quad | \quad -\tilde{\mathbf{C}}\right]$$

  - Left 3x3 submatrix of **P** is product of upper-triangular matrix and orthogonal matrix

Finding Camera Translation

- Find homogeneous coordinates of C in the scene
- **C** is the null vector of matrix **P**
  - **P C = 0**:

$$\mathbf{P} = \mathbf{K}\,\mathbf{R}\left[\mathbf{I}_3 \quad | \quad -\tilde{\mathbf{C}}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & X_c \\ 0 & 1 & 0 & Y_c \\ 0 & 0 & 1 & Z_c \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- Find null vector **C** of **P** using SVD
  - **C** is the unit singular vector of **P** corresponding to the smallest singular value (the last column of **V**, where **P** = **U D V**^{T} is the SVD of **P**)

Finding Camera Orientation and Internal Parameters

- Left 3x3 submatrix **M** of **P** is of form **M=K R**
  - **K** is an upper triangular matrix
  - **R** is an orthogonal matrix
- Any non-singular square matrix **M** can be decomposed into the product of an upper-triangular matrix **K** and an orthogonal matrix **R** using the RQ factorization
  - Similar to QR factorization but order of 2 matrices is reversed

RQ Factorization of $\mathbf{M}$

$$\mathbf{R_x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix}, \mathbf{R_y} = \begin{bmatrix} c' & 0 & s' \\ 0 & 1 & 0 \\ -s' & 0 & c' \end{bmatrix}, \mathbf{R_z} = \begin{bmatrix} c'' & -s'' & 0 \\ s'' & c'' & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Compute $c = -\dfrac{m_{33}}{(m_{32}^2 + m_{33}^2)^{1/2}} \;, \quad s = \dfrac{m_{32}}{(m_{32}^2 + m_{33}^2)^{1/2}}$
- Multiply $\mathbf{M}$ by $\mathbf{R}_x$. The resulting term at (3,2) is zero because of the values selected for c and s
- Multiply the resulting matrix by $\mathbf{R}_y$, after selecting c' and s' so that the resulting term at position (3,1) is set to zero
- Multiply the resulting matrix by $\mathbf{R}_z$, after selecting c'' and s'' so that the resulting term at position (2,1) is set to zero

$$\mathbf{M\,R_x R_y R_z} = \mathbf{K} \Rightarrow \mathbf{M} = \mathbf{K\,R_z^T R_y^T R_x^T} = \mathbf{K\,R}$$

Computing Matrix **P**

- Use corresponding image and scene points
  - 3D points $\mathbf{X_i}$ in world coordinate system
  - Images $\mathbf{x_i}$ of $\mathbf{X_i}$ in image
- Write $\mathbf{x_i} = \mathbf{P}\ \mathbf{X_i}$ for all i
- Similar problem to finding projectivity matrix **H** (i.e. homography) in homework

Improved Computation of P

- $\mathbf{x_i} = \mathbf{P}\,\mathbf{X_i}$ involves homogeneous coordinates, thus $\mathbf{x_i}$ and $\mathbf{P}\,\mathbf{X_i}$ just have to be proportional: $\mathbf{x}_\mathrm{i} \times \mathbf{P}\,\mathbf{X}_\mathrm{i} = 0$
- Let $\mathbf{p}_1{}^\mathrm{T}$, $\mathbf{p}_2{}^\mathrm{T}$, $\mathbf{p}_3{}^\mathrm{T}$ be the 3 row vectors of $\mathbf{P}$

$$\mathbf{P}\,\mathbf{X}_\mathrm{i} = \begin{bmatrix} \mathbf{p}_1^\mathrm{T}\mathbf{X}_\mathrm{i} \\ \mathbf{p}_2^\mathrm{T}\mathbf{X}_\mathrm{i} \\ \mathbf{p}_3^\mathrm{T}\mathbf{X}_\mathrm{i} \end{bmatrix} \qquad \mathbf{x}_\mathrm{i} \times \mathbf{P}\,\mathbf{X}_\mathrm{i} = \begin{bmatrix} v'_\mathrm{i}\,\mathbf{p}_3^\mathrm{T}\mathbf{X}_\mathrm{i} - w'_\mathrm{i}\,\mathbf{p}_2^\mathrm{T}\mathbf{X}_\mathrm{i} \\ w'_\mathrm{i}\,\mathbf{p}_1^\mathrm{T}\mathbf{X}_\mathrm{i} - u'_\mathrm{i}\,\mathbf{p}_3^\mathrm{T}\mathbf{X}_\mathrm{i} \\ u'_\mathrm{i}\,\mathbf{p}_2^\mathrm{T}\mathbf{X}_\mathrm{i} - v'_\mathrm{i}\,\mathbf{p}_1^\mathrm{T}\mathbf{X}_\mathrm{i} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \mathbf{0}_4^\mathrm{T} & -w'_\mathrm{i}\,\mathbf{X}_\mathrm{i}^\mathrm{T} & v'_\mathrm{i}\,\mathbf{X}_\mathrm{i}^\mathrm{T} \\ w'_\mathrm{i}\,\mathbf{X}_\mathrm{i}^\mathrm{T} & \mathbf{0}_4^\mathrm{T} & -u'_\mathrm{i}\,\mathbf{X}_\mathrm{i}^\mathrm{T} \\ -v'_\mathrm{i}\,\mathbf{X}_\mathrm{i}^\mathrm{T} & u'_\mathrm{i}\,\mathbf{X}_\mathrm{i}^\mathrm{T} & \mathbf{0}_4^\mathrm{T} \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = 0 \qquad \begin{bmatrix} \mathrm{p}_1 \\ \mathrm{p}_2 \\ \mathrm{p}_3 \end{bmatrix} \text{is a } 12 \times 1 \text{ vector}$$

Improved Computation of P, cont'd

- Third row can be obtained from sum of u'_i times first row - v'_i times second row

$$\begin{bmatrix} \mathbf{0}_4^T & -w'_i \mathbf{X}_i^T & v'_i \mathbf{X}_i^T \\ w'_i \mathbf{X}_i^T & \mathbf{0}_4^T & -u'_i \mathbf{X}_i^T \\ -v'_i \mathbf{X}_i^T & u'_i \mathbf{X}_i^T & \mathbf{0}_4^T \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = 0$$

- So we get 2 independent equations in 11 unknowns (ignoring scale)
- With 6 point correspondences, we get enough equations to compute matrix **P**

$$\mathbf{A}\,\mathbf{p} = 0$$

Solving **A p** = 0

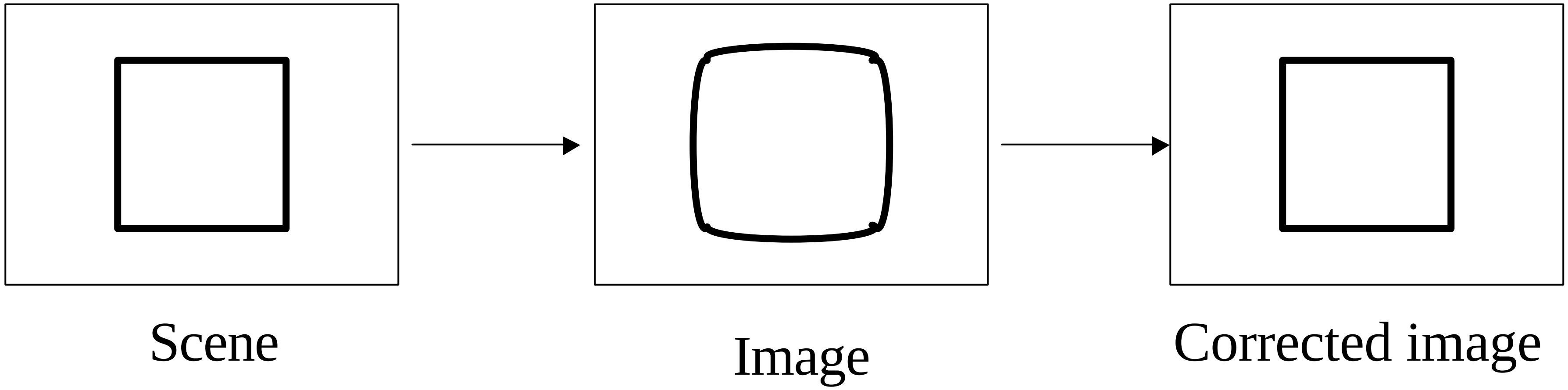
- Linear system **A p** = 0
- When possible, have at least 5 times as many equations as unknowns (28 points)
- Minimize || **A p** || with the constraint || **p** || = 1
  - P is the unit singular vector of **A** corresponding to the smallest singular value (the last column of **V**, where **A = U D V**^{T} is the SVD of **A**)
- Called Direct Linear Transformation (DLT)

Improving **P** Solution with Nonlinear Minimization

- Find **p** using DLT
- Use as initialization for nonlinear minimization of $\sum_{\mathbf{i}} d(\mathbf{x_i}, \mathbf{PX_i})^2$
  - Use Levenberg-Marquardt iterative minimization

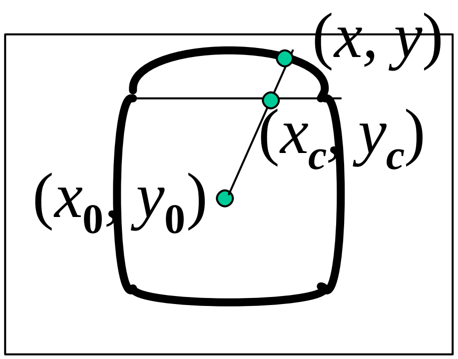
Radial Distortion

- We have assumed that lines are imaged as lines
- Not quite true for real lenses
  - Significant error for cheap optics and for short focal lengths



Radial Distortion Modeling

- In pixel cordinates the correction is written

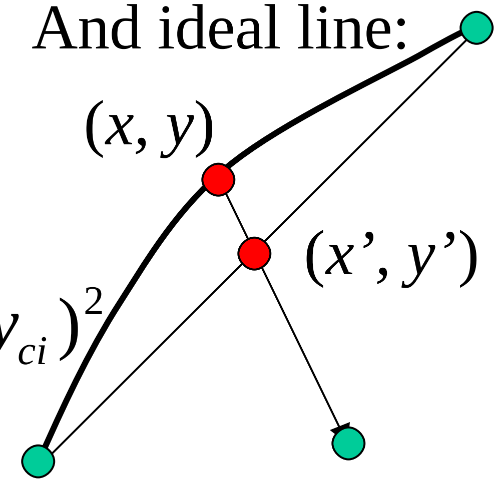


$$x_c - x_0 = L(r)(x - x_0)$$
$$y_c - y_0 = L(r)(y - y_0)$$

with

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$
$$L(r) = 1 + k_1 r + k_2 r^2 + \ldots$$

Distorted line
And ideal line:



- Minimize $f(k_1, k_2) = \sum_i (x'_i - x_{ci})^2 + (y'_i - y_{ci})^2$
  using lines known to be straight
  (x’,y’) is radial projection of (x,y) on straight line

References

- Multiple View Geometry in Computer Vision, R. Hartley and A. Zisserman, Cambridge University Press, 2000, pp. 138-183
- Three-Dimensional Computer Vision: A Geometric Approach, O. Faugeras, MIT Press, 1996, pp. 33-68
- “A Versatile Camera Calibration Technique for 3D Machine Vision”, R. Y. Tsai, IEEE J. Robotics & Automation, RA-3, No. 4, August 1987, pp. 323-344