

## Mensuration Algorithm

**Assumption:** The vanishing line is given

**Problem:** Given the reference length  $\|MN\|=1$  in multiple frames as  $m_i n_i$  and a probe line segment  $ST$  also in multiple frames as  $s_j t_j$ , measure  $\|ST\|$ .

**Fact:** If for all frame  $i$ ,  $M_i$  are same, then  $N_i$  are distributed in a unit circle centering at  $M_i$ . Furthermore, if  $S_j=M_i$ , then  $T_j$  and  $N_i$  are on two concentric circles respectively.

### Solution

Arbitrary select point  $o$  and parallel move  $m_i n_i$  to  $op_i$ ,  $s_j t_j$  to  $oq_j$ . World points  $P_i$  are on a circle  $C_p$  and  $Q_j$  on a concentric circle  $C_q$

$$C_p = C_0 - r_p^2 \quad C_q = C_0 - r_q^2$$

$C_0$  is the circle centering at  $O$  with zero radius.

$$H^{-1} \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ L(1) & L(2) & L(3) \end{pmatrix} \quad L \text{ is the vanishing line}$$

$p_i$  and  $q_j$  are on two concentric ellipses centering at  $o$

$$E_{p,q} = E_0 - r_{p,q}^2 L \quad L = H^{-T} \text{diag}(0,0,1) H^{-1}$$

$$E_0 : a_0(x - x_0)^2 + b_0(x - x_0)(y - y_0) + c_0(y - y_0)^2 = 0$$

Finally  $a_0(x_p - x_0)^2 + b_0(x_p - x_0)(y_p - y_0) + c_0(y_p - y_0)^2 - \alpha_p(Lp_i)^2 = 0$

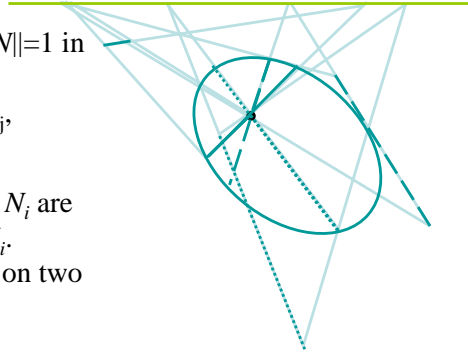
$$a_0(x_q - x_0)^2 + b_0(x_q - x_0)(y_q - y_0) + c_0(y_q - y_0)^2 - \alpha_q(Lq_j)^2 = 0$$

$(a_0 \ b_0 \ c_0 \ \alpha_p \ \alpha_q)^T$  can be solved by Singular Value Decomposition (SVD)

The ratio between the two radii

$$r = \sqrt{\alpha_p / \alpha_q}$$

## Example



## Wheelbase Detection



**Key:** Use the intensity difference between the wheel covers and the tires

## the Vanishing Line Estimation

Represent the vanishing line as  $(-\sin \phi \sin \theta, \cos \phi \sin \theta, -f \cos \theta)$

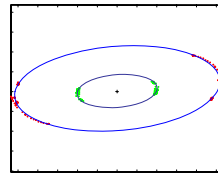
$\theta$ : elevation angle  $\phi$ : rotation angle  $f$ : focal length

Minimize the variance of  $\cos^2(\theta) = \frac{\cos^2 \phi - r^2 \sin^2 \phi - 2c_a \cos \phi \sin \phi + c_a^2 \sin^2 \phi}{\sin^2 \phi - r^2 \cos^2 \phi + c_a \cos \phi \sin \phi + c_a^2 \cos^2 \phi}$

$$r = \left| \frac{\delta x_2 \delta y_1 - \delta x_1 \delta y_2}{\delta x_1^2 - \delta x_2^2} \right| \quad c_a = \frac{\delta x_1 \delta y_1 - \delta x_2 \delta y_2}{\delta y_1^2 - \delta y_2^2} \quad \delta x_i = x_{i1} - x_{i2} \quad \delta y_i = y_{i1} - y_{i2}$$

## Objective & Contribution

An automated system for wheelbase measurement of moving vehicles



## Mensuration Result

