Kanade–Lucas–Tomasi Tracking (KLT tracker)

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Talk Outline

- importance for Computer Vision
- gradient based optimization
- good features to track
- experiments

Importance in Computer Vision

- Firstly published in 1981 as an image registration method [3].
- Improved many times, most importantly by Carlo Tomasi [5, 4]
- Free implementation(s) available¹.
- After more than two decades, a project² at CMU dedicated to this single algorithm and results published in a premium journal [1].
- Part of plethora computer vision algorithms.

¹http://www.ces.clemson.edu/~stb/klt/
²http://www.ri.cmu.edu/projects/project_515.html

Tracking of dense sequences — camera motion
Tracking of dense sequences — object motion

Alignment of an image (patch)

Goal is to align a template image $T(x)$ to an input image $I(x)$. $x$ column vector containing image coordinates $[x, y]^T$. The $I(x)$ could be also a small subwindow within an image.

Original Lucas-Kanade algorithm I

Goal is to align a template image $T(x)$ to an input image $I(x)$. $x$ column vector containing image coordinates $[x, y]^T$. The $I(x)$ could be also a small subwindow within an image.

Set of allowable warps $W(x; p)$, where $p$ is a vector of parameters. For translations

$$W(x; p) = \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix}$$

$W(x; p)$ can be arbitrarily complex

The best alignment minimizes image dissimilarity

$$\sum_x [I(W(x; p)) - T(x)]^2$$
Original Lucas-Kanade algorithm II

\[ \sum_x [I(W(x; p)) - T(x)]^2 \]

is a non-linear optimization! The warp \( W(x; p) \) may be linear but the pixels value are, in general, non-linear. In fact, they are essentially unrelated to \( x \).

It is assumed that some \( p \) is known and best increment \( \Delta p \) is sought. The modified problem

\[ \sum_x [I(W(x; p + \Delta p)) - T(x)]^2 \]

is solved with respect to \( \Delta p \). When found then \( p \) gets updated

\[ p \leftarrow p + \Delta p \]

Original Lucas-Kanade algorithm III

\[ \sum_x [I(W(x; p + \Delta p)) - T(x)]^2 \]

linearized by performing first order Taylor expansion

\[ \sum_x [I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x)]^2 \]

\( \nabla I = \left[ \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right] \) is the gradient image\(^3\) computed at \( W(x; p) \). The term \( \frac{\partial W}{\partial p} \) is the Jacobian of the warp.

\(^3\)As a vector it should have been a column wise oriented. However, for sake of clarity of equations row vector is exceptionally considered here.

Original Lucas-Kanade algorithm IV

Derive

\[ \sum_x [I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x)]^2 \]

with respect to \( \Delta p \)

\[ 2 \sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^\top \left[ I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) \right] \]

setting equality to zero yields

\[ \Delta p = H^{-1} \sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^\top \left[ T(x) - I(W(x; p)) \right] \]

where \( H \) is the Hessian matrix

\[ H = \sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^\top \left[ \nabla I \frac{\partial W}{\partial p} \right] \]
The Lucas-Kanade algorithm—Summary

Iterate:
1. Warp $I$ with $W(x; p)$
2. Warp the gradient $\nabla I$ with $W(x; p)$
3. Evaluate the Jacobian $\frac{\partial W}{\partial p}$ at $(x; p)$ and compute the steepest descent image $\nabla I \frac{\partial W}{\partial p}$
4. Compute the Hessian $H = \sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^T \left[ \nabla I \frac{\partial W}{\partial p} \right]$
5. Compute $\Delta p = H^{-1} \sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^T [T(x) - I(W(x; p))]$
6. Update the parameters $p \leftarrow p + \Delta p$

until $\|\Delta p\| \leq \epsilon$

Example of convergence

Convergence video: Initial state is within the basin of attraction
What are good features (windows) to track?

How to select good templates $T(x)$ for image registration, object tracking.

$$\Delta p = H^{-1} \sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^T [T(x) - I(W(x; p))]$$

where $H$ is the Hessian matrix

$$H = \sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^T \left[ \nabla I \frac{\partial W}{\partial p} \right]$$

The stability of the iteration is mainly influenced by the inverse of Hessian. We can study its eigenvalues. Consequently, the criterion of a good feature window is $\min(\lambda_1, \lambda_2) > \lambda_{\min}$ (texturedness).

What are good features (windows) to track?

Consider translation $W(x; p) = \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix}$. The Jacobian is then

$$\frac{\partial W}{\partial p} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H = \sum_x \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix}^T \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \sum_x \begin{bmatrix} \frac{\partial I}{\partial x}^2 & \frac{\partial I}{\partial x} \frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x} \frac{\partial I}{\partial y} & \frac{\partial I}{\partial y}^2 \end{bmatrix}$$

The image windows with varying derivatives in both directions. Homeogeneous areas are clearly not suitable. Texture oriented mostly in one direction only would cause instability for this translation.
What are the good points for translations?

The Hessian matrix

\[ H = \sum_x \begin{bmatrix} \left( \frac{\partial I}{\partial x} \right)^2 & \frac{\partial^2 I}{\partial x \partial y} \\ \frac{\partial^2 I}{\partial y \partial x} & \left( \frac{\partial I}{\partial y} \right)^2 \end{bmatrix} \]

Should have large eigenvalues. We have seen the matrix already, where?

Harris corner detector [2]!

Experiments - no occlusions

Experiments - occlusions
Experiments - occlusions with dissimilarity

Experiments - object motion

References


