0. **Optical flow**

a) For a point \([x_0, y_0]\) imaged at \([x, y]\), the projection equations are:

\[
\begin{align*}
    x(t) &= \frac{f x_0}{z(t)} \quad \text{and} \quad y(t) &= \frac{f y_0}{z(t)}
\end{align*}
\]

where \(z(t)\) is a function of the distance b/w the camera and the planar surface: \(z(t) = z_0 - Wt\)

**Optical flow components:**

\[
U = \frac{dx(t)}{dt} = \left(\frac{1}{z(t)}\right)^2 \left(-\frac{f x_0}{z_0^2}\right) \frac{dz(t)}{dt}
\]

\[
\frac{dz(t)}{dt} = -W \implies U = \frac{1}{\left(\frac{z(t)}{z_0}\right)^2} \left(-x(t) - \frac{W}{2}\right) (-W)
\]

\[
U = \frac{W x(t)}{z(t)}
\]

\[
V = \frac{W y(t)}{z(t)}
\]

b) **Optical flow is dependent on time (from a))**

c) \(\nabla^2 u = 0 = \nabla^2 v\)

d) **Time of impact:** \(z(t) = z_0 - Wt = 0\)

\[
\frac{x}{t} = \frac{y}{t} \quad \implies \quad t = \left|\frac{x}{u}\right|, \quad t = \left|\frac{y}{v}\right|
\]

t can be found by dividing the image co-ordinate
3) Surfaces, $z_1 = 2(x^2 + 2xy + y^2)$

$$z_2 = (x^2 + 4xy + y^2)$$

If reflectance map is rotationally symmetric, then

$$R(p, q) = f(p^2 + q^2), \quad p = \frac{\partial z(x, y)}{\partial x}, q = \frac{\partial z(x, y)}{\partial y}$$

$z_1$: $p_1 = 2(2x + y), \quad q_1 = 2(x + 2y)$

$z_2$: $p_2 = 2x + 4y, \quad q_2 = 4x + 2y$

$$p_1^2 + q_1^2 = 4 \left[ 4x^2 + y^2 + 4xy + x^2 + 4y^2 + 4xy \right]$$

$$= 4 \left[ 5x^2 + 5y^2 + 8xy \right]$$

$$= p_2^2 + q_2^2$$

4) $R(p_2, q_2) = ap_2 + bq_2 + c; \quad p_2 = \frac{\partial z(x, y)}{\partial x}, q_2 = \frac{\partial z(x, y)}{\partial y}$

$$R(p_{2_1}, q_{2_2}) = a \left( p_2 + b \frac{dq}{ds} \right) + b \left( q_2 + -a \frac{dq}{ds} \right) + c$$

$$= ap_2 + bq_2 + c = R(p_2, q_2)$$

Silhouettes need not be the same, depends on the relation b/w $g(s)$ and the normals to the