Homework 4

1. Consider a camera moving along the optical axis toward a planar surface at right angles to the optical axis.
   a. Show that the optical flow is given by

      \[ U = \frac{Wx}{Z} \quad \text{and} \quad v = \frac{Wy}{Z}, \]

      where \( W \) is the velocity and \( Z \) the distance to the plane.
   b. Is the optical flow stationary (that is independent of time)?
   c. Is the Laplacian of the optical flow zero?
   d. How could you predict the time to impact? (Note that this can be done despite the fact that we cannot recover the absolute value of either height or velocity.)

2. Show that the rotation about an arbitrary axis is equivalent to rotation about an axis through the origin combined with a translation. Show that the compensating translation is equal to the cross-product of a vector from the origin to the axis and the rotation vector.

3. Show that the two surfaces

      \[ z_1 = 2(x^2 + y^2) \quad \text{and} \quad z_2 = (x^2 + 4xy + y^2) \]

      give rise to the same shading near the origin if a rotationally symmetric reflectance map applies.

4. Suppose that the reflectance map is linear in \( p \) and \( q \), so that

      \[ R(p, q) = ap + bq + c. \]

      We have an image, including the silhouette of a simple convex object of shape \( z = f(x, y) \). Show that the surface

      \[ z_1 = f(x, y) + g(bx-ay), \]

      for an arbitrary differentiable function \( g(s) \), will give rise to the same image. Does the surface \( z_1 \) have the same silhouette? Assume that the derivative of \( g \) is bounded.