Quantum approach to Image processing

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Abstract

Quantum computing is a new trend in computation-theory and a quantum mechanical system has several useful properties like Entanglement .
In this paper tried to explain some method and algortithm for image processing that works in a quantum computer and how to profits from advantages of quantum system , and then illustrate several computational experiment in this direction.

1. Introduction

The theory of quantum mechanics was prompted by the failure of classical physics in explaining a number of microphysical phenomena that were observed at the end of nineteenth and early twentieth centuries . Now, quantum mechanics is vital for understanding the physics of solids, lasers, semiconductor and superconductor devices, plasmas, etc.

In recent years, quantum mechanics has been connected with computer science, information theory in communication and digital signal processing . For example, Shor has showed that integer factoring could be done in polynomial time on a quantum computer . One of major applications of Shor's quantum factorization algorithm is to break RSA public key cryptosystems. Thus, developing new computing methods and signal processing algorithms by borrowing from the principle of quantum mechanics is a very interesting and new research topic.

In this paper, several quantum digital image processing algorithms will be presented including image halftoning algorithm and edge detection method . The details will be described in next sections.

1.1. Introduction of Quantum Computation

Quantum computing is a new approach to computation that has the possibility to revolutionize the field of computer science. The late Nobel Prize winning physicist
Richard Feynman, who was interested in using a computer to simulate quantum systems, first investigated using quantum systems to do computation in 1982 . He realized that the classical storage requirements for quantum systems grow exponentially in the number of particles. So while simulating twenty quantum particles only requires storing a million values, doubling this to a forty particle simulation would require a trillion values. Interesting simulations, say using a hundred or thousand particles, would not be possible, even using every computer on the planet. Thus he suggested making computers that utilized quantum particles as a computational resource that could simulate general quantum systems in order to do large simulations, and the
idea of using quantum mechanical effects to do computation was born. The exponential storage capacity, coupled with some spooky effects like quantum entanglement, has led researchers to probe deeper into the computing power of quantum systems. Quantum computing has blossomed over the past 20 years, demonstrating the ability to solve some problems exponentially faster than any current computer could ever
do. The most famous algorithm, the integer-factoring algorithm of Peter Shor, would allow the most popular encryption methods in use today to be cracked easily, if large enough quantum computers can be constructed. Thus the race is on to develop the theory and hardware that would enable quantum computing to become as widespread as PCs are today. Classical computers, which include all current mainstream computers, work on discrete pieces of information, and manipulate them according to rules laid out by John Von Neumann in the 1940's. In honor of his groundbreaking work, current computers are said to run on a "Von Neumann architecture", which is modeled on an abstraction of discrete pieces of information. However, in recent years, scientists have changed from this abstraction of

computing, to realizing that since a computer must ultimately be a physical device, the rules governing computation should be derived from physical law. Quantum mechanics is one of the most fundamental physical theories, and thus was a good choice to study what computational tasks could be physically achieved. This study led to the profound discovery that quantum mechanics allows much more powerful machines than the
Von Neumann abstraction.

1.2. Introduction of Quantum Bit(qbit)

Just as classical bit has state - either 0 or 1 – a qubit also has a state. Two possible states for a qubit are the states |0> and |1>, which as you might guess correspond to thestates 0 and 1 for a classical bit. Notation like '| >' is called the Dirac notation, and we'll beseeing it often, as it's the standard notation for states in quantum mechanics. The difference between bits and qubit can be in a state other than |0> or |1>. It is also possible to form linear combinations of states, often called superpositions

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

The numbers α and β are complex numbers. Put another way, the state of a qubit is avector in a two-dimensional complex vector space. The special states |0> and |1> are known as computational basis states, and form an orthonormal basis for this vector space.

We can examine a bit to determine whether it is in the state 0 or 1 in classical computer.By contrast, when we measure a qubit, we get either the result 0, with probability$|\alpha|^2$, or the result 1, with probability$|\beta|^2$. Naturally, $|\alpha|^2 + |\beta|^2 = 1$. In general a qubit is a unit vector in a two-dimensional complex vector space [3].

We explain the superpositions $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ by analogy with sonic wave as following.

Suppose there are three persons Alice, Bob and you in a closed room. Alice and Bob speak in a sample wave $|A\rangle = f_A(t) = e^{im\omega t}$ and $|B\rangle = f_B(t) = e^{in\omega t}$ respectively, where m ≠ n and they are both integers. We can distinguish Alice from Bob because the two sample waves are orthogonal (i.e., $\langle A|B\rangle = \int_0^{\frac{2\pi}{\omega}} e^{i(n-m)\omega t} dt = 0$). When Alice speak in the closed room, your ears will receive a sonic wave $g_A(t) = I_A e^{\phi_A}|A\rangle$, where IA is the amplitude of the wave and the phase ϕ_A is cause by the distance between Alice and you. If Alice and Bob speak simultaneously, your ears will receive a superposition:

$$|\psi_{AB}\rangle = I_A e^{\phi_A}|A\rangle + I_B e^{\phi_B}|B\rangle$$

Let

$$I = \sqrt{(I_A)^2 + (I_B)^2}$$

$$\alpha_A = \frac{I_A}{I} e^{i\phi_A} \text{ and } \beta_B = \frac{I_B}{I} e^{i\phi_B}$$

Thus,

$$|\psi_{AB}\rangle = I(\alpha_A|A\rangle + \beta_B|B\rangle)$$

Your ears can distinguish Alice's voice from the superposition $|\psi_{AB}\rangle$. That is, '+' implies two sonic wave |A> and |B> exist in the superposition simultaneously and they can be distinguished from the superposition. If Alice speaks very aloud (i.e., $|\alpha_A|^2 \to 1$ or $|\beta_B|^2 \to 0$) , you will always hear Alice's voice. This case is analogous with the case $|\alpha|^2 \to 1$of quantum computation. If $|\alpha|^2 \to 1$, you will get the result 0 always, with probability $|\alpha|^2 \approx 1$. This property is utilized to design quantum algorithm such as Grover's algorithm. You can operate the two distinguished sample wave |A> and |B> simultaneously. For example, you can send the voice $|\psi_{AB}\rangle$in a radio and change Alice's volumes and Bob's volumes
simultaneously by pushing the volume button on radio. That is, performing once operation causes the changing of two sonic waves simultaneously. This case is analogous with quantum parallelism. Fig. 1 shows the analogies between quantum superpositions and sonic wave.

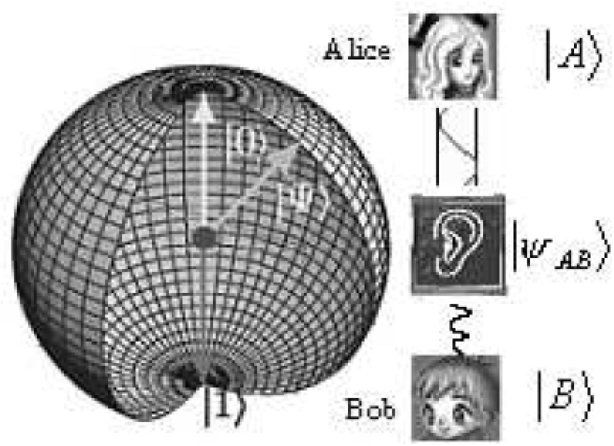


FIG. 1: The schematic diagram of the analogies between quantum superpositions and sonic wave.

1.3. Operation of Computation Acting on Qubit

Classical computer circuits consist of wires and logic gates. The wires are used to carry information around the circuit, while the logic gates perform manipulations of the information, converting it from one form to another. It is the fundamental of classical computation that classical computer circuits can realize the operations of Boolean algebra. For example, classical NOT gate makes 0 and 1 states interchanged. The operations of Boolean algebra can also be realized on quantum computer by utilizing single qubit gates and controlled-NOT gates. For example, quantum NOT gate takes the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to the corresponding state in which the role of |0> and |1> have interchanged. All digital operation can be realized by utilizing unitary operation that is the combination of some single qubit gates and controlled-NOT gates [3].

1.4. Quantum Measurement

Measurement according to the rules of Quantum Mechanics is a non-trivial and highly counter-intuitive process. Firstly, it must be said that the measurement results taken from a quantum system are inherently of a probabilistic nature. In other words, regardless of the carefulness in the preparation of a measurement procedure, the possible outcomes of such measurement will be distributed according to a certain probability distribution.

Secondly, once a measurement has been performed, a quantum system in unavoidably altered due to the interaction with the measurement apparatus. Thus, it makes sense to talk about pre-measurement and post-measurement quantum states for an arbitrary quantum system.

Thirdly, in order to perform a measurement it is needed to define a set of measurement operators. This set of operators must fulfill a number of rules that allows one to compute the actual probability distribution as well as post-measurement quantum states.

In order to clarify these points, let us work out a simple example. Assume we have a polarized photon with associated polarization orientations 'horizontal' and 'ver- tical'. The horizontal polarization direction is denoted by |0> and the vertical polarization direction is denoted by |1>.

Thus, an arbitrary initial state for our photon can be described by the state by $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are complex numbers constrained by the normalization condition $|\alpha|^2 + |\beta|^2 = 1$ and {|0>, |1>} is the computa-tional basis spanning $\mathcal{H}^2$.

Now, let us construct two measurement operators $\hat{M}_0 = |0\rangle\langle 0|$ and $\hat{M}_1 = |1\rangle\langle 1|$ and two measurement out-comes a_0, a_1. Then, the full observable used for measure-ment in this experiment is $\hat{M} = a_0|0\rangle\langle 0| + a_1|1\rangle\langle 1|$.

According to the rules of Quantum Mechanics, the probabilities of obtaining outcome a_0 or outcome a_1 are given by $p(a_0) = |\alpha|^2$ and $p(a_1) = |\beta|^2$. Correspond-ing post-measurement quantum states are as follows: if outcome = a_0 then $|\Psi_{pm}\rangle = |0\rangle$; if outcome = a_1 then $|\Psi_{pm}\rangle = |1\rangle$.

It is possible to construct a full quantum measurement theory for both vector and density matrix representations of quantum systems. Measurement theory and its implications in QC and QIP are open and fruitful fields of research.

1.5. Quantum Entanglement

Suppose we have two qubits, the first in the state $\alpha_0|0\rangle + \alpha_1|1\rangle$ and the second in the state $\beta_0|0\rangle + \beta_1|1\rangle$. What is the joint state of the two qubits? The answer is, the (tensor) product of the two: $\alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle$ Given an arbitrary state of two qubits, can we specify the state of each individual qubit in this way? No, in general the two qubits are *entangled* and cannot be decomposed into the states of the individual qubits. For example, consider the state $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$, which is one of the famous Bell states. It cannot be decomposed into states of the two individual qubits. Entanglement is one of the most mysterious aspects of quantum mechanics and is ultimately the source of the power of quantum computation.

1.6. Quantum Parallelism

Quantum parallelism allows quantum computers to evaluate a function f(x) for many different values of x simultaneously. The power of quantum computation is due to the fact that the state of a quantum computer can be a superposition of basis states and we can perform an operation on multiple quantum states simultaneously. For example, suppose f(x) : {0, 1} → {0, 1} is a function with a one-bit domain and range. We need at least two times calculating for obtain the values f(0) and f(1) on classical computer. For arbitrary function f(x), there is quantum circuit U_f that can transform $|0, y\rangle$ and $|1, y\rangle$ into $|0, y \oplus f(0)\rangle$ and $|1, y \oplus f(1)\rangle$ respectively by performing only one time calculating, where $\oplus$ indicates addition modulo 2. That is, $\frac{|0\rangle|y\rangle + |1\rangle|y\rangle}{\sqrt{2}} \xrightarrow{U_f} \frac{|0\rangle|y \oplus f(0)\rangle + |1\rangle|y \oplus f(1)\rangle}{\sqrt{2}}$

, where '+' implies two states $|0\rangle\,|y\rangle$ and $|1\rangle\,|y\rangle$ exist in the superposition of states simultaneously. The formula implies also that the two states $|0\rangle\,|y\rangle$ and $|1\rangle\,|y\rangle$ are converted to $|0\rangle\,|y \oplus f(0)\rangle$ and $|1\rangle\,|y \oplus f(1)\rangle$ simultaneously. That is, the values f(0) and f(1) are calculated simultaneously [3, 4].

1.7. Introduction Quantum Fourier transform (QFT) , Kelappnecker's DCT and Lattore's Quantum Representation of Image

The quantum Fourier transform (QFT [3,4]) on an orthonormal basis $|0\rangle, |1\rangle, \cdots, |N-1\rangle$ is defined to be a linear operator with the following action on the basis states,

$$QFT|j\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |k\rangle$$

Equivalently, the action on an arbitrary state may be written

$$QFT(\sum_{j=0}^{N-1} \alpha_j |j\rangle) = \sum_{j=0}^{N-1} \alpha_j (QFT|j\rangle)$$

Similar to other unitary operation, QFT is unitary operation that only acts on basis states. It's an error opinion that unitary operation can act on coefficients of basis states. Figure 2 illustrates that QFT only acts on basis states |0> and |1>.

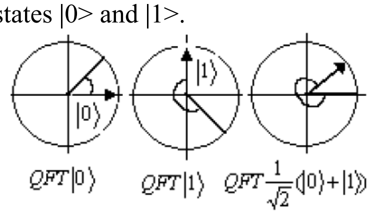


FIG. 2: QFT only acts on basis states |0> and |1>: QFT $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(QFT|0\rangle + QFT|1\rangle)$.

Klappenecker presents a method to realize DCT of types I, II, III, and IV on a quantum computer by utilizing QFT [5].

Define DCT of types I as [5]

$$C_N^I := \sqrt{(\frac{2}{N})}[\alpha_i \cos \frac{ij\pi}{N}]_{i,j=0\ldots N}$$

The discrete sine transforms (DST) of types I denoted by S_N^I is defined accordingly. [5]

Let discrete Fourier transform (DFT) be [5]

$$F_N = \frac{1}{\sqrt{N}}[\sum_{k=0}^{N-1} (e^{2\pi ijk/N})]_{k,l=0..N-1}$$

Let [5]

$$T_N = \begin{pmatrix} 1 & & & & & & & & \\ & \frac{1}{\sqrt{2}} & & & & & \frac{i}{\sqrt{2}} & & \\ & & \ddots & & & & & \ddots & \\ & & & \frac{1}{\sqrt{2}} & & & & & \frac{i}{\sqrt{2}} \\ & & & & 1 & & & & \\ & & & \frac{1}{\sqrt{2}} & & & & & -\frac{i}{\sqrt{2}} \\ & & \ddots & & & & & \ddots & \\ & \frac{1}{\sqrt{2}} & & & & & -\frac{i}{\sqrt{2}} & & \end{pmatrix}$$

Klappenecker's DCT derives from QFT and depends on QFT. Indeed, the DST and DCT can be recovered from the DFT by a base change

$$T_N^+ \cdot F_{2N} \cdot T_N = C_N^I \oplus iS_N^I$$

Since efficient quantum circuit for the DFT (i.e., QFT) are known, it remains to find an efficient implementation of the matrix TN. A quantum circuit is proposed by Klappenecker to realize the matrix TN. This is the primitive idea of Klappenecker's DCT [5].

The result of QFT or Klappenecker's DCT seems to indicate that quantum computer can be used to very quickly compute the Fourier transform, which would be fantastically useful in a wide range of applications. However, that is not exactly the case; the Fourier transform or Klappenecker's DCT is being performed on the information 'hidden' in the amplitudes of the quantum state. This information is not directly accessible to measurement. The catch, of course, is that if the output state is measured, it will collapse each qubit into the state |0> or |1>, preventing us from learning the transform result directly[3]. In addition, the contents of section 2.1 in this paper shows that Klappenecker's DCT cannot be applied to DCT of image compression too. Klappenecker's DCT is useful on many other signal processing maybe.

Latorre presentes a novel quantum representation for image compression [9]. The entropy of images is, in general, very large. An image with large entropy is hard to compress. The idea in Latorre's paper is to keep the largest eigenvalues of the Schmidt decompositions when the picture is written in a renormalization group manner, that is, the largest contribution to the entropy in that basis. Latorre's algorithm is nice but it is not competitive with jpeg. The reason is that jpeg uses details of how human see. The quantization table used in jpeg is tailored to the human eye. Unless there are quantum quantization methods are incorporated in Latorre's algorithm, there is no way it is as efficient as jpeg.

2. THE REPRESENTATION OF IMAGE BY USING QUANTUM STATES.

2.1. Classical Data structure of 1-D DCT

For a given vector $x = (x_0, x1, \cdots, x_{N-1})$, we can declare a BYTE array "BY TE x[N]" to store it by using c language, where c language is compiler language of classical computer [7]:

$$x[0] = x_0, x[1] = x_1, ..., x[N-1] = x_{N-1}$$

There is a logical mapping to associate subscript with component of vector x:

$$Mapping : i \longmapsto x[i] \quad (0 \leq i < N) \qquad (1)$$

The logical mapping is necessary because it associate data with the corresponding logical address. CPU accesses value x[i] according to the subscript i (i.e., logical address)

The mapping is done by memory-management unit (MMU) of Operating System [8]. The operation of access data is a very very fast operation so that the time of access can be ignored when designing algorithm.

Fig3 illustrates the logical mapping. Fig4 illustrates the physical realization of the logical mapping [8].

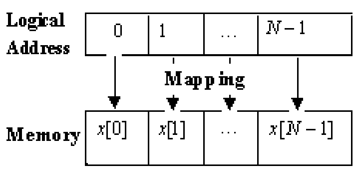


FIG. 3: The Conception of the Logical Mapping. The mapping associates data with the corresponding logical address

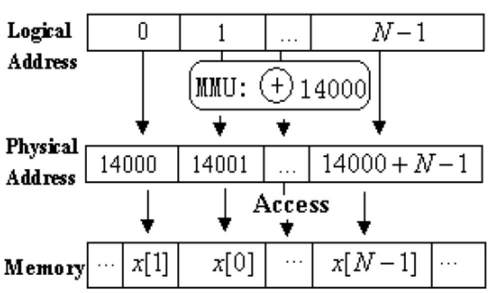


FIG. 4: The Illustration of the Physical Realization of the Logical Mapping [8]. Accessing data is very very fast operation so that the time of access can be ignored when designing algorithm.

Similar to vector x, the vectors $\overrightarrow{f_i}$, $\overrightarrow{C}$, $\overrightarrow{D_u}$, matrix D and matrix F can be stored in array respectively, and the Operating System of classical computer will establish the mapping (equation 1) automatically [8].

For example, we declared a two dimensional array "BY TE arrayImage[N][N]" to store matrix F. The mapping between position (i, j) and pixel value f_{ij} is defined as

$$Mapping : (i, j) \mapsto arrayImage[i][j] = f_{ij} \qquad (2)$$

The above mapping (equations 1 and 2) should be also kept in quantum computation so that arbitrary component of vector or matrix is associated with the corresponding subscript.

By the definition of DCT , QFT and Klappenecker's DCT cannot both keep the mapping. Therefore, More suitable quantum data structure is required in order to keep the mapping and harness the power of quantum computation for image compression.

2.2.The Quantum Representation of Image

2.2.1. Data Structure of Quantum Representation of Image

To keep the mapping in equations (1) and (2), the following database technique is presented to represent image F in this paper:

First, all vectors $\overrightarrow{f_i} = (f_{0i}, f_{1i}, ..., f_{(N-1)i})^T \ (0 \leq i < N)$ are stored in a database. Each vector is regard as a record with unique index i.

Second, all vectors are loaded into CPU simultaneously and form the superposition of States $\frac{1}{\sqrt{N}}|ancilla1\rangle(\sum_{i=0}^{N-1}|i\rangle|\overrightarrow{f_i}\rangle)|ancilla2\rangle$ by using quantum addressing scheme and unitary operation LOAD.

LOAD operation that is denoted by UL is defined as

$$|i\rangle|0\rangle\cdots|0\rangle \xrightarrow{U_L} |i\rangle|0 \oplus f_{0i}\rangle\cdots|0 \oplus f_{(N-1)i}\rangle$$

, where $\oplus$ denotes addition modulo 2, that can be realized by utilizing controlled NOT operation [3].

In vector notation,

$$|i\rangle|0\rangle \overset{U_L}{\longmapsto} |i\rangle|0 \oplus \overrightarrow{f_i}\rangle \qquad (3)$$

LOAD operation is the basic operation of quantum computer ([3], chapter 6).

Figure 5 illustrates the representation of image by using quantum states.

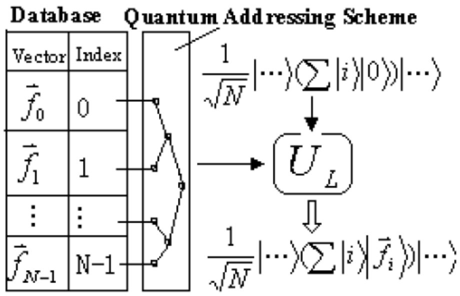


FIG. 5: The Representation of Image by Using Quantum States: LOAD operation $U_L : |i\rangle|0\rangle \overset{U_L}{\mapsto} |i\rangle|0 \oplus \vec{f_i}\rangle$ is a CNOT operation and is a very very fast operation so that the time of addressing can be ignored when designing quantum algorithm such as Grover's algorithm. It is clear that the most efficient possible algorithm is in this model of computation

The proposed representation of image in this paper keep the mapping in equation (2) so that subscript(j, i) is associated with corresponding pixel value f_{ji}. State $\frac{1}{\sqrt{N}}|ancilla1\rangle(\sum_{i=0}^{N-1}|i\rangle|\vec{f_i}\rangle)|ancilla2\rangle$ is entanglement state when ancilla1 and ancilla2 are constants. Therefore, if we obtain value i from the second register, we will get the unique mapping vector $\vec{f_i}$ in third register. Thus, the mapping is kept.

3.Quantum Mechanics and QSP Framework

Quantum mechanics is the basis for an understanding of quantum signal processing (QSP), so we first provide the necessary background of quantum mechanics in this section. For simplicity, let us study the simplest quantum system known as the "square well", which is a particle in a one-dimensional box. The Schrodinger's equation of this system is given by

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial^2 x} + V(x)\psi$$

where *m* is the mass of the particle, *h* is the Plank's constant, and potential function $V(x)=0$ for $0<x<L$ and $V(x)=\infty$ otherwise, as shown in Fig.1. Thus, the boundary conditions of probability wave function ψ are $\psi(0,t)=0$ and $\psi(L,t)=0$. Solving this differential equation, we get

$$\psi(x,t) = \sum_n \alpha_n\psi_n(x,t)$$

Where $\sum_n|\alpha_n|^2 = 1$ and

$$\psi_n(x,t) = \sqrt{\frac{2}{L}}\sin(k_n x)e^{\frac{-i}{\hbar}E_n t}$$

with $k_n = \frac{n\pi}{L}$ and $E_n = \frac{\pi^2\hbar^2}{2mL^2}n^2$. Because *n* is integer, the energy level has been quantized into discrete valve. Moreover, it is not difficult to verify that the complex valued function $\psi_n(x,t)$ form an orthonormal set. If we only consider the two lowest energy levels, the wave function of interest $\psi(x,t)$ is

$$\psi(x,t) = \alpha_1\sqrt{\frac{2}{L}}\sin(k_1 x)e^{\frac{-i}{\hbar}E_1 t} + \alpha_2\sqrt{\frac{2}{L}}\sin(k_2 x)e^{\frac{-i}{\hbar}E_2 t}$$

Define $|0\rangle = \sqrt{\frac{2}{L}}\sin(k_1x)$, $|1\rangle = \sqrt{\frac{2}{L}}\sin(k_2x)$, $a(t) = \alpha_1 e^{\frac{-i}{\hbar}E_1 t}$

And $b(t) = \alpha_2 e^{\frac{-i}{\hbar}E_2 t}$, then $\psi(x,t)$ can be rewritten as

$$\psi(x,t) = a(t)|0\rangle + b(t)|1\rangle$$

Thus, at position x, we can write our state vector abstractly as

$$|\psi(t)\rangle = [a(t) \quad b(t)]^T$$

Where $|a(t)|^2 + |b(t)|^2 = 1$. This two-level system represents a quantum bit or qubit. Based on the above example, the four postulates of quantum mechanics are described as follows:

Postulate 1: State space

Associate to any isolated physical system is complex vector space with inner product (i.e., a Hilbert space) known as the state space of the system. The system is completely described by its state vector $|\psi(t)\rangle$, which is a unit vector in the system's state space.

Postulate 2: Time evolution

The time evolution of the state of a closed system is described by the Schrodinger equation or a unitary transformation. That is, the state $|\psi(t_1)\rangle$ is related to the state $|\psi(t_2)\rangle$ by a unitary operator *U*. In our qubit example, this means

$$\begin{bmatrix} a(t_2) \\ b(t_2) \end{bmatrix} = U \begin{bmatrix} a(t_1) \\ b(t_1) \end{bmatrix}$$

Postulate 3: Measurement

Quantum measurement are described by a collection of matrices $\{M_m\}$ which satisfy the complete equation $\sum_m M_m^H M_m = I$ and $M_m^2 = M_m$, where *H* denotes transpose conjugate and *I* is an identity matrix. The probability that measurement outcomes *m* occurs is given by

$$p(m) = \langle \psi(t) | M_m^H M_m | \psi(t) \rangle$$

where notation $\langle \psi(t) |$ denotes the transpose conjugate of $|\psi(t)\rangle$. And, the state of the system after measurement is

$$\frac{M_m |\psi(t)\rangle}{\sqrt{\langle \psi(t) | M_m^H M_m | \psi(t) \rangle}} |m\rangle$$

In our qubit example, we choose the measurement matrices as

$$M_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, the probability $p(0)=|a(t)|^2$ and $p(1)=|b(t)|^2$. The state after measurement in this example is

$$\frac{M_0|\psi(t)\rangle}{|a(t)|}|0\rangle = \frac{a(t)}{|a(t)|}|0\rangle, \quad \frac{M_1|\psi(t)\rangle}{|b(t)|}|1\rangle = \frac{b(t)}{|b(t)|}|1\rangle$$

Note that measurement consistency is a fundamental postulate of quantum mechanics, i.e., repeated measurements on a system must yield the same outcome. This result is valid under the condition $M_m^2 = M_m$. Therefore, the state of the system after a measurement must be such that if we re-measure the system in this state, then the final state after this second measurement will be identical to the state after the first measurement.

Postulate 4: Composite system
The state space of a composite physical system is tensor product of the state spaces of the component physical system. As an example, let two quantum states be $|x\rangle = a_1|0\rangle + b_1|1\rangle$ and $|y\rangle = a_2|0\rangle + b_2|1\rangle$, then the tensor product of $|x\rangle$ and $|y\rangle$ is given by

$$|x\rangle \otimes |y\rangle = a_1 b_1 |0\rangle \otimes |0\rangle + a_1 b_2 |0\rangle \otimes |1\rangle + a_2 b_1 |1\rangle \otimes |0\rangle + a_2 b_2 |1\rangle \otimes |1\rangle$$

We often use the abbreviated notation $|xy\rangle$ for the tensor product $|x\rangle \otimes |y\rangle$. Thus, we have

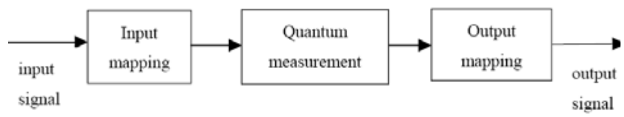
$$|xy\rangle = a_1 b_1 |00\rangle + a_1 b_2 |01\rangle + a_2 b_1 |10\rangle + a_2 b_2 |11\rangle$$

Note that the postulate 4 can not enable us to obtain the following two qubit state

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

This sate is the well-known entangled state. Entanglement has played a crucial role in quantum computation and quantum information.

Based on the above four postulates of quantum mechanics, quantum computation algorithm (QCA), quantum information theory (QIT) and quantum signal processing (QSP) can be developed. In this paper, we will focus on the QSP framework proposed by Eldar and Oppenheim [13]. The general quantum signal processing framework is shown below.



Three steps involved are input mapping, QSP measurement and output mapping. First, the input scalar value of signal is first mapped into the linear combination of state vectors of a quantum system. Then, we measure the superposition state vectors using quantum measurement postulate. Finally, the measurement outcome is mapped to the algorithm output. In the following, we will use this QSP framework to develop three quantum image processing algorithms. These algorithms are derived by using physical quantum phenomena and constraints as metaphors.

3.1.Quantum Image Halftoning Algorithm

Digital image halftoning techniques have been widely used in the printing of books, magazines, newspapers and in computer printers. It transforms grayscale images into bi-level image before output device actually displays or prints out the image. Because the human eyes possess the ability to integrate the neighboring halftone dots, human eyes will perceive them as continuous-tone images. So far, the popular halftoning algorithms are error diffusion method and ordered dither method [10][11]. Given a gray scale image x(m,n) with size MxN, the quantum algorithm to obtain the halftoning image y(m,n) is described below:

3.1.1. Input mapping

In this stage, we will transform each pixel x(m,n) into a quantum bit |q(m,n)> which is a superposition of two quantum states |0> and |1>, i.e.,

$$|q(m,n)\rangle = c_0|0\rangle + c_1|1\rangle$$

where $|c_0|^2$ is the measurement probability of state |0> and $|c_1|^2$ is the measurement probability of state |1>. Thus, we have the relation $|c_0|^2+|c_1|^2=1$. If each pixel value x(m,n) has been normalized into the range [0,1], then |c0|2 and |c1|2 can be computed by following method. Like error diffusion, the probability that y(m,n)=1 will depends on its foregoing neighboring output values and coming neighboring input pixel values. Let us define two sums as

$$s_1 = \sum_{k=-1}^{1}\sum_{l=0}^{1} x(m-k,n-l) + x(m,n+1) + x(m,n+2)$$

$$s_2 = y(m-1,n-1) + y(m-1,n) + y(m,n-1)$$

then the value *P* is calculated by

$$P = \frac{s_1 - s_2}{5}$$

Thus, the measurement probabilities $|c_0|^2$ and $|c_1|^2$ can be computed by using the following two equations:

$$|c_1|^2 = f(P) \qquad |c_0|^2 = 1 - f(P)$$

where function f(P) is

$$f(P) = \frac{1}{1 + e^{-(P-a)/b}}$$

When states |0> and |1> correspond to the basis vectors (1,0) and (0,1) in two dimensional space, then the mapping quantum bit |q(m,n)> corresponds to the vector (1-f(P),f(P)). Thus, each pixel value of image is mapped into a vector in two dimensional space.

3.1.2. QSP measurement

The measurement postulate of quantum mechanics says that when we measure a superimposed quantum bit, it will be projected into one of the states allowed by the measurement. Thus, after quantum bit |q(m,n)> is measured, the measurement outcome |o(m,n)> is either state |0> or |1>. The measurement has a probability of $|c_0|^2$ of being found in state |0>, and a probability of $|c_1|^2$ of state |1>. In this paper, the measurement is performed as follows: First, we generate a random number z uniformly distributed in the range [0,1] per each measurement of qubit |q(m,n)>. Then, if z is in the range [0, $|c_1|^2$], then outcome |o(m,n)> is the state |1>. Moreover, if z is in the range ($|c_1|^2$,1], then outcome |o(m,n)> is the state |0>.

3.1.3. Output mapping

The output pixel value y(m,n) of halftoning image is determined from the measurement outcome |o(m,n)> by the using the following rule: If the outcome |o(m,n)> is the state |0>, then y(m,n)=0. And, if the outcome |o(m,n)> is the state |1>, then y(m,n)=1. After the output mapping, bi-level image y(m,n) is the final desired halftoning image.

3.1.4. Experimental Result

In this experiment, the input gray level image x(m,n) is the Lena with size 256x256, as shown in Fig.6(a). Fig.6(b)-(e) shows the halftoning images by using binary threshold method, ordered dither method, error diffusion method and proposed method. The output image of binary threshold method is computed by

$$y(m,n) = \begin{cases} 1 & x(m,n) \geq 0.5 \\ 0 & otherwise \end{cases}$$

The details of ordered dither method and error diffusion method can be referred to [6]. The parameters *a* and *b* of proposed method is *a=0.5* and *b=0.05*. From the results in Fig.6, we see that the uniformity of image of error diffusion is better than other three methods. However, the contrast of image of proposed method is superior to those of other three methods because the edges are sharper.

3.2.Quantum Image Edge Detection Algorithm

The local discontinuities in an image luminance from one level to another are called edge. The edge detection is a problem of fundamental importance in image analysis [1][12]. Edges characterize object boundaries and are therefore useful for segmentation and registration of objects in scenes. Given a gray scale image x(m,n) with size MxN, the quantum algorithm to detect its edges is described below:

3.21. Input mapping

In this stage, the pixel x(m,n) is transformed into a quantum bit $|q(m,n)\rangle = c_0|0\rangle + c_1|1\rangle$. Three steps to compute the state probabilities $|c_0|^2$ and $|c_1|^2$ is given as follows: First, the row derivative gr(m,n) and column derivative gc(m,n) are computed by using the Sobel operator below:

$$g_r(m,n)=[x(m+1,n-1)+2x(m+1,n)+x(m+1,n+1)]$$
$$-[x(m-1,n-1)-2x(m-1,n)-x(m-1,n+1)]$$

$$g_c(m,n)=[x(m-1,n+1)+2x(m,n+1)+x(m+1,n+1)]$$
$$-[x(m-1,n-1)-2x(m,n-1)-x(m+1,n-1)]$$

Second, the magnitude of gradient vector (gr(m,n), gc(m,n)) can be computed by

$$g(m,n)= [g_r(m,n)^2 + g_c(m,n)^2]^{1/2}$$

Finally, the probabilities $|c_0|^2$ and $|c_1|^2$ are determined by the equations:

$$|c_1|^2 = f(g(m,n))$$

$$|c_0|^2 = 1-f(g(m,n))$$

where the function *f(.)* is defined by

$$f(x) = \frac{1}{1 + e^{-(x-a)/b}}$$

The parameters *a* and *b* are two prescribed positive numbers. From this mapping, it is clear that the larger gradient vector magnitude g(m,n) is, the larger value of probability $|c_1|^2$ has.

3.2.2. QSP measurement

When quantum bit |q(m,n)> is measured, the measurement outcome |o(m,n)> is either state |0> or |1>. In this paper, the measurement is performed as follows: First, we generate a random number z uniformly distributed in the range [0,1] per each measurement of qubit |q(m,n)>. Then, if z is in the range [0, $|c_1|^2$], then outcome |o(m,n)> is the state |1>. Moreover, if z is in the range $|c_1|^2$,1], then outcome |o(m,n)> is the state |0>.

3.2.3. Output mapping

The output pixel value y(m,n) of edge image is determined from the measurement outcome |o(m,n)> by the using the following rule: If the outcome |o(m,n)> is the state |0>, then y(m,n)=0. And, if the outcome |o(m,n)> is the state |1>, then y(m,n) may be 1 or 0. When following two cases hold, then y(m,n)=1. Otherwise, y(m,n)=0.

Case 1: If four conditions |o(m,n)>=|1>, |gr(m,n)|> |gc(m,n)|, g(m,n)>g(m+1,n) and g(m,n)>g(m-1,n) hold simultaneously, then output y(m,n)=1.

Case 2: If four conditions |o(m,n)>=|1>, |gc(m,n)|> |gr(m,n)|, g(m,n)>g(m,n+1) and g(m,n)>g(m,n-1) hold simultaneously, then output y(m,n)=1.

After the output mapping, bi-level image y(m,n) is the final desired edge image.

3.2.4. Experimental Result

In this experiment, the input gray level image x(m,n) is the Lena with size 256x256, as shown in Fig.7(a). Fig.7(b)-(e) shows the edge images by using proposed method, Sobel method, Laplacian of Gaussian (log) method and Canny method. These edge images are obtained by using Matlab instructions listed below:

EDGE(x,'sobel',10)

EDGE(x,'log',4,1)

EDGE(x,'canny',0.1)

The parameters *a* and *b* of proposed method is *a=70* and *b=0.1*. From the results in Fig.7, we see that the proposed method is almost comparable with the Sobel method because Sobel mask is used to estimate the gradient vector.

5.Illustrations, photographs

Figure 6 The image halftoning results. (a) original image (b) binary threshold method (c) ordered dithering (d) error diffusion (e) proposed method.

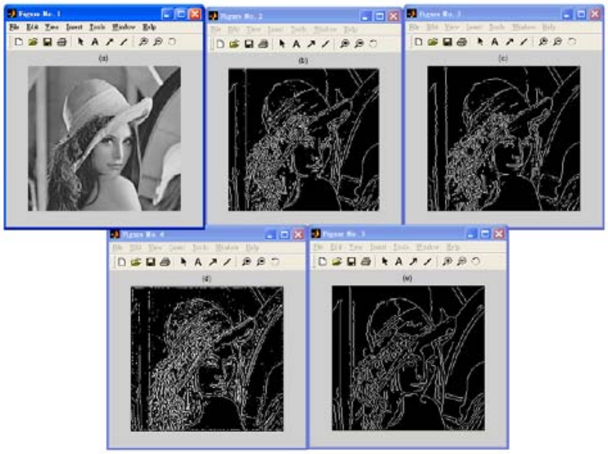


Figure 7 The image edge detection results. (a) original image (b) proposed method (c) Sobel method (d) Laplacian of Gaussian method (e) Canny method.

References

[1] Rafael C. Gonzalez, Richard E. Woods, ”Digital Image Processing,” Publishing House of Electronics Industry ; Prentice Hall (Beijin, 2002)

[2] Chao-Yang Pang, Zheng-Wei Zhou, Ping-Xing Chen, and Guang-Can Guo, ”Design of Quantum VQ Iteration and Quantum VQ Encoding Algorithm Taking O(√N) Steps for Data Compression,” Chinese Physics. (It will published on No. 15, Vol. 3, 2006 and will appear on March, 2006)

[3] M. A. Nielsen and I. L. Chuang, ”Quantum Computationand and Quantum Information,” Cambridge University Press, Cambridge, 2000

[4] A. Galindo and M. A. Mart´ın-Delgado, ”Information and computation: Classical and quantum aspects,” Reviews of Modern Physics, vol.74, April 2002

[5] A. Klappenecker and M. R¨otteler, ”Discrete Cosine Transforms on Quantum Computers,” Proceedings of the 2nd International Symposium on Image and Signal Processing and Analysis, Pula, Croatia, June 19-21, 2001, S. Loncaric, H. Babic (eds.), pages 464-468, IEEE, 2001 (arXiv:quant-ph/0111038)

[6] Eli Biham, et al., “Grover’s quantum search algorithm for an arbitrary initial amplitude distribution,” Phys. Rev. A 60, pp.2742-2745 , 1999

[7] AI Kelley and Ira Pohl, ”Programming in C (Fourth Edition),” China Machine Press, China, 2004

[8] Abraham Silberschatz, Peter Baer Galvin, ”Operating System Concepts (Fifth Edition),” John Wiley & Sons, Inc.,1997.

[9] Jose I. Latorre, ”Image compression and entanglement,” arXiv:quant-ph/0510031

[10] D. E. Knuth, Digital halftones by dot diffusion, ACM Trans. on Graphics, vol.6, no.4, pp.245-273, Oct. 1987.

[11] D.L. Lau and G. R. Arce, *Modern Digital halftoning*, Marcel Dekker, 2001.

[12] W. K. Pratt, Digital Image Processing, Third Edition, John Wiley & Sons, 2001.

[13] Y. C. Eldar and A.V. Oppenheim, *Quantum Signal Processing*, IEEE Signal Processing Magazine, pp.12-32, Nov. 2002.