Problem 1.4.3 Solution

The first generation consists of two plants each with genotype yg or gy. They are crossed to produce the following second generation genotypes, $S = \{yy, yg, gy, gg\}$. Each genotype is just as likely as any other so the probability of each genotype is consequently 1/4. A pea plant has yellow seeds if it possesses at least one dominant y gene. The set of pea plants with yellow seeds is

$$Y = \{yy, yg, gy\}$$

So the probability of a pea plant with yellow seeds is

$$P[Y] = P[yy] + P[yg] + P[gy] = 3/4.$$

Problem 1.5.4 Solution

Define D as the event that a pea plant has two dominant y genes. To find the conditional probability of D given the event Y, corresponding to a plant having yellow seeds, we look to evaluate

$$P[D/Y] = P[DY] / P[Y]$$

Note that $P[DY]$ is just the probability of the genotype yy. From Problem 1.4.3, we found that with respect to the color of the peas, the genotypes yy, yg, gy, and gg were all equally likely. This implies

$$P[DY] = P[yy] = 1/4 \qquad P[Y] = P[yy, gy, yg] = 3/4$$

Thus, the conditional probability can be expressed as

$$P[D/Y] = P[DY] / P[Y] = (1/4)/(3/4) = 1/3$$

Problem 1.6.4 Solution

(a) Since $A \cap B = \emptyset$, $P[A \cap B] = 0$. To find $P[B]$, we can write

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

$$5/8 = 3/8 + P[B] - 0$$

Thus, $P[B] = 1/4$. Since A is a subset of B^c, $P[A \cap B^c] = P[A] = 3/8$. Furthermore, since A is a subset of B^c, $P[A \cup B^c] = P[B^c] = 3/4$.

(b) The events A and B are dependent because

$$P[AB] = 0 \ != \ 3/32 = P[A]\,P[B]$$ (‘!=’ means not equal to)

(c) Since C and D are independent $P[CD] = P[C]P[D]$. So

$$P[D] = P[CD]\,/\,P[C] = (1/3)/\,(½) = 2/3$$

In addition, $P[C \cap D^c] = P[C] - P[C \cap D] = 1/2 - 1/3 = 1/6$. To find $P[C^c \cap D^c]$, we first observe that

$$P[C \cup D] = P[C] + P[D] - P[C \cap D] = 1/2 + 2/3 - 1/3 = 5/6$$

By De Morgan’s Law, $C^c \cap D^c = (C \cup D)^c$. This implies

$$P[C^c \cap D^c] = P[(C \cup D)^c] = 1 - P[C \cup D] = 1/6$$

Note that a second way to find $P[C^c \cap D^c]$ is to use the fact that if C and D are independent, then C^c and D^c are independent. Thus

$$P[C^c \cap D^c] = P[C^c]\,P[D^c] = (1 - P[C])(1 - P[D]) = 1/6$$

Finally, since C and D are independent events, $P[C/D] = P[C] = 1/2$.

(d) Note that we found $P[C \cup D] = 5/6$. We can also use the earlier results to show

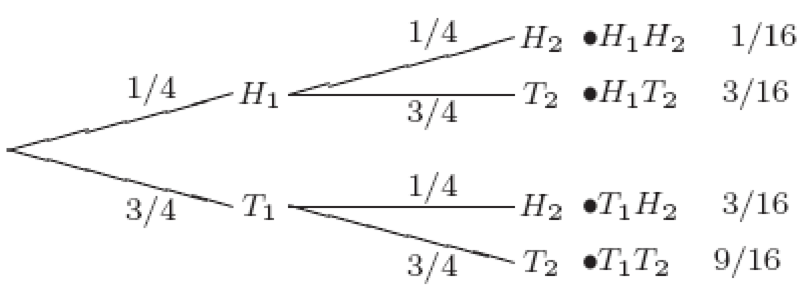
$$P[C \cup D^c] = P[C] + P[D] - P[C \cap D^c] = 1/2 + (1 - 2/3) - 1/6 = 2/3$$

(e) By Definition 1.7, events C and D^c are independent because

$$P[C \cap D^c] = 1/6 = (1/2)(1/3) = P[C]\,P[D^c]$$

Problem 1.7.1 Solution

A sequential sample space for this experiment is



(a) From the tree, we observe

$$P[H_2] = P[H_1H_2] + P[T_1H_2] = 1/4.$$

This implies

$$P[H_1/H_2] = P[H_1H_2]\,/\,P[H_2] = (1/16)\,/\,(1/4) = 1/4$$

(b) The probability that the first flip is heads and the second flip is tails is $P[H_1T_2] = 3/16$.