Ans(1-1)

Let x_1 = left-weight = 2.93
x_2 = left-distance = 2.94
x_3 = right-weight = 3.70
x_4 = right-distance = 2.40

To determine the class in terms of x_1, x_2, x_3 and x_4, we need to evaluate the conditional probabilities below:

$$\mathrm{P}\{\mathrm{L} \mid x_1 = 2.93,\ x_2 = 2.94,\ x_3 = 3.70,\ x_4 = 2.40\}$$
$$\mathrm{P}\{\mathrm{B} \mid x_1 = 2.93,\ x_2 = 2.94,\ x_3 = 3.70,\ x_4 = 2.40\}$$
$$\mathrm{P}\{\mathrm{R} \mid x_1 = 2.93,\ x_2 = 2.94,\ x_3 = 3.70,\ x_4 = 2.40\}$$

The maximum probability will determine the class value corresponding to the given input values.
Take $\mathrm{P}\{\mathrm{L} \mid x_1 = 2.93,\ x_2 = 2.94,\ x_3 = 3.70,\ x_4 = 2.40\}$ as an example,
$\mathrm{P}\{\mathrm{L} \mid x_1 = 2.93,\ x_2 = 2.94,\ x_3 = 3.70,\ x_4 = 2.40\}$

$$= \frac{P\{x_1 = 2.93, x_2 = 2.94, x_3 = 3.70, x_4 = 2.40|L\} \cdot P\{L\}}{P\{x_1 = 2.93, x_2 = 2.94, x_3 = 3.70, x_4 = 2.40\}} \text{ (Bayes' Theorem)}$$

$$= \frac{P\{x_1 = 2.93|L\} \cdot P\{x_2 = 2.94|L\} \cdot P\{x_3 = 3.70|L\} \cdot P\{x_4 = 2.40|L\} \cdot P\{L\}}{P\{x_1 = 2.93, x_2 = 2.94, x_3 = 3.70, x_4 = 2.40\}}$$

Similarly for class B and R.
$P\{L\} = 0.46$, $P\{B\} = 0.08$, $P\{R\} = 0.46$

Ans(1-1)

Since the denominators are the same, we can ignore them and just calculate the numerators using the normal distribution. According to the weka results, the conditional probabilities can be approximated as follows:

$$P\{x_1 = 2.93|L\} = \frac{1}{\sqrt{2\pi}\sigma_1} \exp \frac{(2.93-\mu_1)^2}{2{\sigma_1}^2}, \mu_1 \text{ is } 3.6111 \text{ and } \sigma_1 \text{ is } 1.2254.$$

$$P\{x_2 = 2.94|L\} = \frac{1}{\sqrt{2\pi}\sigma_2} \exp \frac{(2.94-\mu_2)^2}{2{\sigma_2}^2}, \mu_2 \text{ is } 3.6111 \text{ and } \sigma_2 \text{ is } 1.2254.$$

$$P\{x_3 = 3.70|L\} = \frac{1}{\sqrt{2\pi}\sigma_3} \exp \frac{(3.70-\mu_3)^2}{2{\sigma_3}^2}, \mu_3 \text{ is } 2.3993 \text{ and } \sigma_3 \text{ is } 1.3295.$$

$$P\{x_4 = 2.40|L\} = \frac{1}{\sqrt{2\pi}\sigma_4} \exp \frac{(2.40-\mu_4)^2}{2{\sigma_4}^2}, \mu_3 \text{ is } 2.3993 \text{ and } \sigma_3 \text{ is } 1.3295.$$

$$P\{x_1 = x|B\} = P\{x_2 = x|B\} = P\{x_3 = x|B\} = P\{x_4 = x|B\} = \frac{1}{\sqrt{2\pi}\sigma} \exp \frac{(x-\mu)^2}{2\sigma^2}, \mu \text{ is } 2.9388 \text{ and } \sigma \text{ is } 1.4056.$$

$$P\{x_1 = 2.93|R\} = \frac{1}{\sqrt{2\pi}\sigma_1} \exp \frac{(2.93-\mu_1)^2}{2{\sigma_1}^2}, \mu_1 \text{ is } 2.3993 \text{ and } \sigma_1 \text{ is } 1.3295.$$

$$P\{x_2 = 2.94|R\} = \frac{1}{\sqrt{2\pi}\sigma_2} \exp \frac{(2.94-\mu_2)^2}{2{\sigma_2}^2}, \mu_2 \text{ is } 2.3993 \text{ and } \sigma_2 \text{ is } 1.3295.$$

$$P\{x_3 = 3.70|R\} = \frac{1}{\sqrt{2\pi}\sigma_3} \exp \frac{(3.70-\mu_3)^2}{2{\sigma_3}^2}, \mu_3 \text{ is } 3.6111 \text{ and } \sigma_3 \text{ is } 1.2254.$$

$$P\{x_4 = 2.40|R\} = \frac{1}{\sqrt{2\pi}\sigma_4} \exp \frac{(2.40-\mu_4)^2}{2{\sigma_4}^2}, \mu_3 \text{ is } 3.6111 \text{ and } \sigma_3 \text{ is } 1.2254.$$

Ans(1-1)

$$
\begin{aligned}
&P\{x_1 = 2.93|L\} \cdot P\{x_2 = 2.94|L\} \cdot P\{x_3 = 3.70|L\} \cdot P\{x_4 = 2.40|L\} \cdot P\{L\} \\
&= 2.79027 \times 10^{-4} \cdot 2.80285 \times 10^{-4} \cdot 1.85875 \times 10^{-4} \cdot 3.00069 \times 10^{-4} \cdot 0.46 \\
&= 2.00653 \times 10^{-15}
\end{aligned}
$$

$$
\begin{aligned}
&P\{x_1 = 2.93|B\} \cdot P\{x_2 = 2.94|B\} \cdot P\{x_3 = 3.70|B\} \cdot P\{x_4 = 2.40|B\} \cdot P\{B\} \\
&= 2.83819 \times 10^{-4} \cdot 2.83823 \times 10^{-4} \cdot 2.45065 \times 10^{-4} \cdot 2.63755 \times 10^{-4} \cdot 0.08 \\
&= 4.16544 \times 10^{-16}
\end{aligned}
$$

$$
\begin{aligned}
&P\{x_1 = 2.93|R\} \cdot P\{x_2 = 2.94|R\} \cdot P\{x_3 = 3.70|R\} \cdot P\{x_4 = 2.40|R\} \cdot P\{R\} \\
&= 2.77049 \times 10^{-4} \cdot 2.76210 \times 10^{-4} \cdot 3.24696 \times 10^{-4} \cdot 1.99847 \times 10^{-4} \cdot 0.46 \\
&= 2.28417 \times 10^{-15}
\end{aligned}
$$

So, the given input belongs to class R since its corresponding probability is higher than each of the other two classes (L and B).

Ans(1-2)

The conditioanl probabilities are shown as below:

$$P\{x_1|L\}, P\{x_2|L\} = \begin{cases} \frac{61}{290} = 0.21 & \text{if } x_1 < 2.5 \\ \frac{229}{290} = 0.79 & \text{if } x_1 > 2.5 \end{cases}$$

$$P\{x_3|L\}, P\{x_4|L\} = \begin{cases} \frac{170}{290} = 0.59 & \text{if } x_1 < 2.5 \\ \frac{120}{290} = 0.41 & \text{if } x_1 > 2.5 \end{cases}$$

$$P\{x_1|B\}, P\{x_2|B\} = P\{x_3|B\} = P\{x_4|B\} = \begin{cases} \frac{22}{51} = 0.43 & \text{if } x_1 < 2.5 \\ \frac{29}{51} = 0.57 & \text{if } x_1 > 2.5 \end{cases}$$

$$P\{x_1|R\}, P\{x_2|R\} = \begin{cases} \frac{170}{290} = 0.59 & \text{if } x_1 < 2.5 \\ \frac{120}{290} = 0.41 & \text{if } x_1 > 2.5 \end{cases}$$

$$P\{x_3|R\}, P\{x_4|R\} = \begin{cases} \frac{61}{290} = 0.21 & \text{if } x_1 < 2.5 \\ \frac{229}{290} = 0.79 & \text{if } x_1 > 2.5 \end{cases}$$

Ans(1-2)

$$
\begin{aligned}
&P\{x_1 = 2.93|L\} \cdot P\{x_2 = 2.94|L\} \cdot P\{x_3 = 3.70|L\} \cdot P\{x_4 = 2.40|L\} \cdot P\{L\} \\
&= 0.79 \cdot 0.79 \cdot 0.41 \cdot 0.59 \cdot 0.46 \\
&= 6.945 \times 10^{-2}
\end{aligned}
$$

$$
\begin{aligned}
&P\{x_1 = 2.93|B\} \cdot P\{x_2 = 2.94|B\} \cdot P\{x_3 = 3.70|B\} \cdot P\{x_4 = 2.40|B\} \cdot P\{B\} \\
&= 0.57 \cdot 0.57 \cdot 0.57 \cdot 0.43 \cdot 0.08 \\
&= 6.371 \times 10^{-3}
\end{aligned}
$$

$$
\begin{aligned}
&P\{x_1 = 2.93|R\} \cdot P\{x_2 = 2.94|R\} \cdot P\{x_3 = 3.70|R\} \cdot P\{x_4 = 2.40|R\} \cdot P\{R\} \\
&= 0.41 \cdot 0.41 \cdot 0.79 \cdot 0.21 \cdot 0.46 \\
&= 2.789 \times 10^{-2}
\end{aligned}
$$

So, the input belongs to class L having higher probability than the other two classes (R and B).

Ans(1-2)

Starting at the root, we use left-weight first to determine which branch to use. Since x1 = 2.93 is greater than 2.5, you go right to the node that contains left-distance and determine the branch again. With x2 = 2.94, you go right to a leaf node containing class L. As a result, this input is classified as belonging to class L. The results obtained from NaiveBayes and J48graft are the same.