Autoencoders

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SLIDES ADAPTED FROM IAN GOODFELLOW
Problems of Autoencoders

- Unsupervised
  - Lots of data
  - Need priors / regularization

- Probabilistic loss function
  - does not work well for discrete data (more later)
  - hard to explain hidden layer probabilistically
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- So let’s use variational inference
Loss Function

\[ \ell_i \equiv -\mathbb{E}_{z \sim q_\theta(z|x_i)} \left[ \log p_\phi(x_i | z) \right] + \text{KL}(q_\theta(z | x_i) \| p(z)) \]  \hspace{1cm} (1)

- Reconstruction error
- Variational representation distribution
- Regularization
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**Interpretation**

- Lower bound on reconstruction of decoder
- Keep representation constrained
- Probabilistic parameterization
Make this Concrete

- \( \text{KL}(q_\theta(z|x_i) \| p(z)) \)
- \( q(z|x_i) \): normal distribution with output of NN as mean [variational distribution]
- \( p(z) \): standard normal distribution
- Decoder \( p_\phi(x|z) \) depends on model / data:
  - Grayscale Image? Bernoulli distribution for each pixel
  - Words? Multinomial over vocabulary
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Variational Inference Story

\[ \ell_i(\theta) = \mathbb{E}_{q_{\theta}(z|x_i)} \left[ \log p_{\phi}(x_i | z) \right] - \text{KL}(q_{\theta}(z | x_i) \| p(z)) \]  

- Want to optimize \( p_{\phi}(x | z) \) (likelihood)
- ELBO remains lower bound
- Difference is KL between variational distribution and \( p(z) \)
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- Actually simpler than LDA
  - No global latent variables (only \( z \))
  - Can minibatch the data
Variational Inference Story

$$\ell_i(\theta) = \mathbb{E}_{q_\theta(z|x_i)}[\log p_\phi(x_i|z)] - KL(q_\theta(z|x_i) \| p(z))$$  \hspace{1cm} (2)

- Want to optimize $p_\phi(x|z)$ (likelihood)
- ELBO remains lower bound
- Difference is KL between variational distribution and $p(z)$
- Actually simpler than LDA
  - No global latent variables (only $z$)
  - Can minibatch the data
  - But what about $\phi$? (encoder)
Variational EM

- Learn variational parameters
- Update $\phi$ using supervised backprop
Variational EM

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- What if $x$ is discrete? (Later)