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1 Default logic

1.1 Default rules

- Background language, logical closure
- Rules of form $X \rightarrow Y$
- Where $\delta = X \rightarrow Y$, have $Premise(\delta) = X$, $Conclusion(\delta) = Y$. Also, if $S$ set of defaults, have $Conclusion(S) = \{Conclusion(\delta) : \delta \in S\}$
- Priorities

1.2 Fixed priority default theories

- Definition 1 (Fixed priority default theories) A fixed priority default theory $\Delta$ is a structure of the form $\langle W, D, < \rangle$, in which $W$ is a set of ordinary formulas, $D$ is a set of default rules, and $<$ is a strict partial ordering on $D$.

- Definition 2 (Extensions) Let $\Delta = \langle W, D, < \rangle$ be a fixed priority default theory. Then $E$ is an extension of $\Delta$ just in case, for some proper scenario $S$ based on this theory,

$$E = Th(W \cup Conclusion(S)).$$

1.3 Stability

- Definition 3 (Triggered defaults) Let $\Delta = \langle W, D, < \rangle$ be a fixed priority default theory, and $S$ a scenario based on this theory. Then the defaults from $D$ that are triggered in the context of the scenario $S$ are those belonging to the set

$$Triggered_{W,D}(S) = \{\delta \in D : W \cup Conclusion(S) \vdash Premise(\delta)\}.$$

- Definition 4 (Conflicted defaults) Let $\Delta = \langle W, D, < \rangle$ be a fixed priority default theory, and $S$ a scenario based on this theory. Then the defaults from $D$ that are conflicted in the context of the scenario $S$ are those belonging to the set

$$Conflicted_{W,D}(S) = \{\delta \in D : W \cup Conclusion(S) \vdash \neg Conclusion(\delta)\}.$$
• Definition 5 (Defeated defaults: preliminary definition) Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory, and \( S \) a scenario based on this theory. Then the defaults from \( D \) that are defeated in the context of the scenario \( S \) are those belonging to the set

\[
\text{Defeated}_{W,D,<}(S) = \{ \delta \in D : \text{there is a default } \delta' \in \text{Triggered}_{W,D}(S) \text{ such that}
\]
\[
(1) \ \delta < \delta',
(2) \ W \cup \{ \text{Conclusion}(\delta') \} \vdash \neg \text{Conclusion}(\delta)\}.
\]

• Definition 6 (Binding defaults) Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory, and \( S \) a scenario based on this theory. Then the defaults from \( D \) that are binding in the context of the scenario \( S \) are those belonging to the set

\[
\text{Binding}_{W,D,<}(S) = \{ \delta \in D : \delta \in \text{Triggered}_{W,D}(S), \delta \notin \text{Conflicted}_{W,D}(S), \delta \notin \text{Defeated}_{W,D,<}(S) \}.
\]

• Definition 7 (Stable scenarios) Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory, and \( S \) a scenario based on this theory. Then \( S \) is a stable scenario based on the theory \( \Delta \) just in case

\[
S = \text{Binding}_{W,D,<}(S).
\]

1.4 Proper scenarios and extensions

• Definition 8 (Approximating sequences) Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory and \( S \) a scenario based on this theory. Then \( S_0, S_1, S_2, \ldots \) is an approximating sequence that is based on the theory \( \Delta \) and constrained by the scenario \( S \) just in case

\[
S_0 = \emptyset,
S_{i+1} = \{ \delta : \delta \in \text{Triggered}_{W,D}(S_i), \delta \notin \text{Conflicted}_{W,D}(S), \delta \notin \text{Defeated}_{W,D,<}(S) \}.
\]

• Definition 9 (Proper scenarios) Let \( \Delta \) be a default theory and \( S \) a scenario based on this theory, and let \( S_0, S_1, S_2, \ldots \) be an approximating sequence that is based on \( \Delta \) and constrained by \( S \). Then \( S \) is a proper scenario based on \( \Delta \) just in case \( S = \bigcup_{i \geq 0} S_i \).
• **Theorem 1** Let $\Delta = \langle W, D, < \rangle$ be a fixed priority default theory and $S$ a proper scenario based on this theory. Then $S$ is also a stable scenario based on the theory $\Delta$.

• **Theorem 2** A fixed priority default theory $\Delta = \langle W, D, < \rangle$ has an inconsistent extension just in case $W$ is inconsistent.

• **Theorem 3** If a fixed priority default theory has an inconsistent extension, this is its only extension.

• **Theorem 4** Let $S$ and $R$ be proper scenarios based on a fixed priority default theory, with $R \subseteq S$. Then $R = S$.

• **Theorem 5** Let $E$ be an extension of the fixed point default theory $\Delta = \langle W, D, < \rangle$, and suppose $A \subseteq W$. Then $E$ is is also an extension of the theory $\Delta' = \langle W \cup A, D, < \rangle$.

1.5 Some consequence relations

• **Definition 10 (Credulous consequence)** Let $\Delta$ be a default theory. Then $Y$ is a credulous consequence of $\Delta$—written, $\Delta \models_C Y$—just in case $Y \in E$ for some extension $E$ of $\Delta$.

• **Definition 11 (Skeptical consequence)** Let $\Delta$ be a default theory. Then $Y$ is a skeptical consequence of $\Delta$—written, $\Delta \models_S Y$—just in case $Y \in E$ for each extension $E$ of $\Delta$.

• Note that credulous consequence is crazy, in the epistemic case.

• **Observation 1**

If $\langle W, D, < \rangle \models_S A$ and $\langle W \cup \{A\}, D, < \rangle \models_S B$, then $\langle W, D, < \rangle \models_S B$.

1.6 Defeasible arguments

• **Definition 12 (Defeasible arguments)** Where $S$ is a set of default rules and $W$ is a set of propositions, a defeasible argument, originating from $W$ and constructed from $S$, is a sequence of propositions $X_1, X_2, \ldots, X_n$ such that each member $X_i$ of the
sequence satisfies one of the following conditions: (1) \( X_i \) is an axiom of propositional logic; (2) \( X_i \) belongs to \( W \); (3) \( X_i \) follows from previous members of the sequence by modus ponens; or (4) there is some default \( \delta \) from \( S \) such that \( \text{Conclusion}(\delta) \) is \( X_i \) and \( \text{Premise}(\delta) \) is a previous member of the sequence.

- **Definition 13 (Argument extensions)** Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory. Then \( \Phi \) is an argument extension of \( \Delta \) just in case, for some proper scenario \( S \) based on this theory,

\[
\Phi = \text{Argument}_W(S).
\]

- **Definition 14 (Grounded scenarios)** Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory and \( S \) a scenario based on this theory. Then \( S \) is grounded in the theory \( \Delta \) just in case \( \text{Th}(W \cup \text{Conclusion}(S)) \subseteq \text{Conclusion}(\text{Argument}_W(S)) \).

- **Theorem 6** Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory and \( S \) a proper scenario based on this theory. Then \( S \) is also grounded in the theory \( \Delta \).

- **Theorem 7** Let \( \Delta = \langle W, D, < \rangle \) be a fixed priority default theory. Then \( S \) is a proper scenario based on the theory \( \Delta \) just in case \( S \) is both stable and also grounded in this theory.

### 1.7 Reiter default theories

- A **Reiter default** is a rule of the form \( (A : C / B) \).

- If \( \delta \) is the Reiter default above, then \( \text{Premise}(\delta) = A, \text{Conclusion}(\delta) = B, \text{Justification}(\delta) = C \).

- **Definition 15 (Reiter default theories)** A Reiter default theory \( \Delta \) is a structure of the form \( \langle W, D \rangle \), in which \( W \) is a set of ordinary formulas and \( D \) is a set of Reiter default rules.

- **Definition 16 (R-conflicted defaults)** Let \( \Delta = \langle W, D \rangle \) be a Reiter default theory, and \( S \) a scenario based on this theory. Then the defaults from \( D \) that are R-conflicted
in the context of the scenario $S$ are those belonging to the set

$$R\text{-conflicted}_{W,D}(S) = \{ \delta \in D : W \cup \text{Conclusion}(S) \vdash \neg \text{Justification}(\delta) \}.$$ 

- **Definition 17 (Approximating sequences)** Let $\Delta = \langle W, D, < \rangle$ be a Reiter default theory and $S$ a scenario based on this theory. Then $S_0, S_1, S_2, \ldots$ is an approximating sequence that is based on the theory $\Delta$ and constrained by the scenario $S$ just in case

  $$S_0 = \emptyset,$$
  $$S_{i+1} = \{ \delta : \delta \in \text{Triggered}_{W,D}(S_i), \delta \notin R\text{-conflicted}_{W,D}(S) \},$$

- Not all Reiter default theories have proper scenarios, and so not all have extensions.

### 1.8 Normal default theories

- A normal default is a default of the form $A \rightarrow B$.

- A normal default can also be identified with a Reiter default of the form $(A : B / B)$. If the default $\delta$ is normal, then $\text{Justification}(\delta) = \text{Conclusion}(\delta)$.

- **Definition 18 (Normal default theories)** A normal default theory can be defined as either (A) a prioritized default theory whose priority ordering is empty, or as (B) a Reiter default theory containing only normal defaults.

- **Theorem 8** Every normal default theory has an extension.