



# Modifying the reason model

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## Abstract

In previous work, I showed how the “reason model” of precedential constraint could naturally be generalized from the standard setting in which it was first developed to a richer setting in which dimensional information is represented as well. Surprisingly, it then turned out that, in this new dimensional setting, the reason model of constraint collapsed into the “result model,” which supports only a fortiori reasoning. The purpose of this note is to suggest a modification of the reason model of constraint that distinguishes it from the result model even in the dimensional setting.

**Keywords** Precedent · Constraint · Dimensions

## 1 Introduction

In previous work (Horty 2019), I showed how two models of precedential constraint could be generalized from the standard setting in which they were first developed, allowing only CATO-style (Aleven 1997) legal factors, to a richer setting in which dimensional information can be represented as well. The first of these was the *result model* of constraint, supporting only a fortiori reasoning. The second was the *reason model*, supporting a stronger notion of precedential constraint that allows the reasons behind decisions to be taken into account.

These two models lead to distinct notions of constraint in the standard setting, but as shown in Horty (2019), in the context of the dimensional setting, the reason model collapses into the result model, leading to exactly the same notion of constraint. Although surprised by this result, I was willing to accept it, and explored what might be characterized as “pragmatic” means of differentiating the two models, rather than altering the definition of the reason model itself. A number of others, however—especially Bench-Capon and Atkinson (2017a, b, 2018) and Rigoni (2018)—criticized this approach, arguing that the collapse of the reason model into the result model shows

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that the reason model itself must be modified, and mapping out alternative approaches of their own.

I still think there is something to be said for the original definition of the reason model.<sup>1</sup> But I also think the criticisms of Atkinson, Bench-Capon, Rigoni and others show that the results of this original definition are, at time, sufficiently unintuitive that modifications should be considered. The purpose of this note is to propose what I take to be, if not the right modification, then at least the modification that is most in keeping with the motivation underlying the original definition.

Organization: Sect. 2 reviews basic concepts from the dimensional setting. Section 3 presents the original definition of the reason model within this setting, and Sect. 4 presents my proposed modification.<sup>2</sup>

## 2 Basic concepts and notation

In contrast to a *factor*, which can be defined as a legally significant proposition that either holds or does not hold in any given situation, but always favors the same side of a dispute when it does hold, a *dimension* is an ordered set of legally significant values, where the ordering among values reflects the extent to which that value favors one side or another.

The idea can be illustrated with a hypothetical example from Prakken and Sartor (1998) concerning the issue whether an individual who has spent time in another country has changed fiscal domicile with respect to income tax. Among the considerations bearing on this issue is the duration of the individual's stay abroad, where greater duration provides stronger support for change of domicile. It is natural to represent duration through a dimension that can take on a variety of values—the individual might have stayed abroad for a week, a month, six months, a year, five years, and so on. Of course, there may be more than one dimension to consider in a given dispute. In the present case, another relevant dimension is the proportion of the individual's income derived from organizations based abroad, with particular percentages as values and larger percentages favoring change of domicile.

To represent information like this, we postulate a set  $D = \{d_1, d_2, \dots, d_n\}$  of dimensions relevant to some area of dispute. We take  $\pi$  and  $\delta$  to represent the two sides of a given dispute, plaintiff and defendant; and where  $s$  is one of these sides, we let  $\bar{s}$  represent the other, so that  $\bar{\pi} = \delta$  and  $\bar{\delta} = \pi$ . For each dimension from  $D$ , we assume an ordered set of values, ranging from those favoring the side  $s$  to those favoring the side  $\bar{s}$ . Where  $p$  and  $q$  are particular values along some fixed dimension, we take the statement

$$p \preceq^s q$$

<sup>1</sup> See, in particular, the discussion of Kobe Bryant and Isiah Thomas from Sect. 4.2 of Horty (2019).

<sup>2</sup> Note that, in order for the current paper to be self-contained, Sects. 2 and 3 review material already presented in Horty (2019). Readers who are familiar with this material can skip to Sect. 4, though I would suggest spending some time on the example from Fig. 1 and the surrounding discussion.

to mean that the assignment of the value  $q$  to this dimension favors the side  $s$  at least as strongly as the assignment of  $p$ . The ordering on dimension values is assumed to satisfy the partial-order conditions of reflexivity, transitivity, and antisymmetry

$$\begin{aligned} p &\preceq^s p, \\ p &\preceq^s q \text{ and } q \preceq^s r \text{ implies } p \preceq^s r, \\ p &\preceq^s q \text{ and } q \preceq^s p \text{ implies } p = q, \end{aligned}$$

as well as a duality condition

$$p \preceq^s q \text{ if and only if } q \preceq^{\bar{s}} p,$$

according to which  $q$  favors the side  $s$  at least as much as  $p$  just in case  $p$  favors the opposing side  $\bar{s}$  at least as much as  $q$ .

This notation can be illustrated with our fiscal domicile example if we imagine that the plaintiff is an individual’s native country, which is arguing against change of domicile in order to tax that individual’s income, and that the defendant is the individual, who is arguing for change of domicile in order to pay, let us suppose, the lower tax rate available in a foreign country. Here, two possible values along the dimension representing the period of residence abroad are six months and eighteen months. If these values are represented simply as 6 and 18, we have  $18 \preceq^{\pi} 6$ , since the shorter period abroad favors the plaintiff’s argument against change of domicile; duality then tells us that  $6 \preceq^{\delta} 18$ , since the longer period abroad favors the defendant’s argument in favor of change.

Where  $p$  is a value along the dimension  $d$ , the pair  $\langle d, p \rangle$  is a *value assignment*, according to which the dimension  $d$  takes on the value  $p$ . Where  $D$  is the set of dimensions relevant to some domain of dispute, then, a *fact situation*

$$X = \{\langle d, p \rangle : d \in D\}$$

can be defined as a set of values assignments, one for each dimension from  $D$ , subject to the condition that if  $\langle d, p \rangle$  and  $\langle d, p' \rangle$  both belong to  $X$ , then  $p = p'$ . A fact situation, in other words, is a function mapping each dimension to a value along that dimension. We take  $X(d)$  as the value assigned to the dimension  $d$  in the fact situation  $X$ , where this idea is defined in the usual way:

$$X(d) = p \text{ if and only if } \langle d, p \rangle \in X.$$

To illustrate: if  $D = \{d_1, d_2\}$  is the set of dimensions bearing on the question whether an individual changed residence for tax purposes, where  $d_1$  represents length of time abroad and  $d_2$  represents proportion of income earned abroad, then  $X_1 = \{\langle d_1, 30 \rangle, \langle d_2, 60 \rangle\}$  is the fact situation presented by an individual who has spent two and a half years, or thirty months, abroad, while earning sixty percent of his or her income abroad. We would therefore have  $X_1(d_1) = 30$  and  $X_1(d_2) = 60$ .

What makes reasoning in the dimensional setting so much more difficult than reasoning with ordinary factors is that the values assigned to particular dimensions do

not, intrinsically, favor one side or another. There is nothing about the period of thirty months, for example, that says that this length of time abroad should count either in favor of, or against, change of domicile—one court might feel that thirty months favors change because it is longer than a year, while another feels that it tells against change because it is shorter than five years. The idea underlying the current account is that, when confronted with a particular situation, a court might focus on certain *reference values* along one or more dimensions, and then justify its decision by comparing the values of the current situation along those dimensions to the selected reference values. Suppose, for instance, that the value of one year seems, to the court, like a sufficient length of time to justify change of fiscal domicile. The court could then find for the defendant, seeking change, and justify its decision on the grounds that the period abroad lasted at least a year.

This proposition—to spell it out, that the actual period of residence abroad favors the defendant at least as much as a period of one year—is a kind of factor: it either holds or does not hold in any fact situation, and always favors the same side, the defendant, when it does hold. Generalizing from our example, where  $p$  is some value along the dimension  $d$ , we define a *magnitude factor favoring the side  $s$*  as a statement of the form

$$M_{d,p}^s$$

carrying the meaning: the actual value assigned to the dimension  $d$  favors the side  $s$  as least as strongly as the reference value  $p$ . The magnitude factor at work in our example can now be expressed as  $M_{d_1,12}^\delta$ , the proposition that the actual value assigned to  $d_1$ , representing length of time abroad, in the situation at hand favors the defendant at least as much as a value of twelve months, or one year.

Since the value assigned to the dimension  $d$  in some situation  $X$  is simply  $X(d)$ , and since this value favors the side  $s$  at least as strongly as the value  $p$  whenever  $p \preceq^s X(d)$ , the conditions under which a dimensional fact situation satisfies a magnitude factor can be defined as follows:

**Definition 1 (Factor satisfaction)** Where  $X$  is a fact situation and  $M_{d,p}^s$  is a magnitude factor,  $X$  satisfies  $M_{d,p}^s$ —written,  $X \models M_{d,p}^s$ —if and only if  $p \preceq^s X(d)$ .

Returning to our example, we have  $X_1 \models M_{d_1,12}^\delta$ , since  $X_1(d_1) = 30$  and since  $12 \preceq^\delta 30$ —that is, a period abroad of thirty months favors change of domicile, and so the defendant, at least as much as a period of twelve months.

A *reason favoring the side  $s$*  can be defined as a set of magnitude factors favoring  $s$ . A factor collection of the form  $\{M_{d,p}^s, M_{d',q}^s\}$ , then, would be a reason favoring the side  $s$ , carrying the conjunctive meaning that the actual value assigned to the dimension  $d$  favors  $s$  as least as strongly as  $p$  and the actual value assigned to the dimension  $d'$  favors  $s$  as least as strongly as  $q$ . A reason containing only a single factor is identified in meaning with the factor it contains.<sup>3</sup>

<sup>3</sup> A discussion of some of the issues surrounding the identification of sets of magnitude factors with reasons can be found in Sect. 3.1 of Horty (2019).

The notion of satisfaction is lifted to reasons, or sets of factors, by stipulating that a situation satisfies a set of factors whenever it satisfies each factor from that set:

**Definition 2** (Reason satisfaction) Where  $X$  is a fact situation and  $W$  is a reason,  $X$  satisfies  $W$ —written,  $X \models W$ —if and only if  $X$  satisfies each factor contained in  $W$ .

And then one reason can be said to entail another whenever any situation that satisfies the first also satisfies the second:

**Definition 3** Reason entailment Where  $W$  and  $Z$  are reasons,  $W$  entails  $Z$ —written,  $W \Vdash Z$ —if and only if  $X \models Z$  whenever  $X \models W$ , for any fact situation  $X$ .

The entailment relation corresponds to a strength ordering among reasons favoring the same side according to which, where  $W$  and  $Z$  are reasons supporting a particular side,  $W$  is at least as strong as  $Z$  for that side just in case  $W \Vdash Z$ . To illustrate: Because  $\{M_{d_1,30}^\delta\} \Vdash \{M_{d_1,12}^\delta\}$ , the reason  $\{M_{d_1,30}^\delta\}$  is at least as strong as  $\{M_{d_1,12}^\delta\}$  for the defendant—spending thirty months or more abroad favors change of domicile at least as strongly as spending twelve months or more abroad. Because  $\{M_{d_1,12}^\delta, M_{d_2,50}^\delta\} \Vdash \{M_{d_1,12}^\delta\}$ , the reason  $\{M_{d_1,12}^\delta, M_{d_2,50}^\delta\}$  is at least as strong as  $\{M_{d_1,12}^\delta\}$ —spending twelve months or more abroad while earning fifty percent or more of income abroad favors change of domicile at least as strongly as spending twelve months or more abroad.

A rule  $r$  can be defined as a statement of the form  $W \rightarrow s$ , where  $W$  is a reason supporting the side  $s$ . We introduce two functions—*Premise* and *Conclusion*—picking out the premise and the conclusion of a rule, so that, in the case of this particular rule  $r$  we would have  $Premise(r) = W$  and  $Conclusion(r) = s$ . A rule like this is to be interpreted as defeasible, telling us that its premise entails its conclusion, not as a matter of necessity, but only by default. A case can then be defined as a fact situation together with an outcome and a rule justifying this outcome. Such a case, then, is a triple of the form  $c = \langle X, r, s \rangle$ , where  $X$  is a fact situation,  $r$  is the rule of the case, and  $s$  is its outcome. We introduce three additional functions—*Facts*, *Rule*, and *Outcome*—mapping cases into their component parts, so that, in the case  $c$  above, for example, we have  $Facts(c) = X$ ,  $Rule(c) = r$ , and  $Outcome(c) = s$ . The concept of a case is subject to two coherence conditions: first, that the rule of the case must actually apply to the underlying fact situation, or equivalently, that the fact situation satisfies the reason that forms the premise of that rule, and second, that the conclusion of the case rule must match the outcome of the case itself.

These ideas can be illustrated with the case  $c_1 = \langle X_1, r_1, s_1 \rangle$ , where  $X_1 = \{\langle d_1, 30 \rangle, \langle d_2, 60 \rangle\}$  is the underlying dimensional fact situation, where  $r_1$  is the rule  $\{M_{d_1,12}^\delta\} \rightarrow \delta$ , and where  $s_1$  is  $\delta$ , a decision for the plaintiff. This case, then, is one in which, confronted with an individual who has spent two and a half years abroad and during that period earned sixty percent of his or her income abroad, the court ruled for change of fiscal domicile, and so in favor of the defendant, on the grounds that the individual spent at least a year abroad.

Finally, we can define a case base  $\Gamma$  as a set of cases.

### 3 The reason model

The key idea underlying the reason model of precedential constraint is that what matters about a precedent case is the previous court's assessment of the relative weight, or priority, of the conflicting reasons presented by that case; later courts are then constrained simply to reach decisions that are consistent with the priority ordering among reasons already established by cases within a background case base. But how is a priority ordering among reasons determined by a decision in a particular case, and what does it mean for a new decision to be consistent with that ordering?

We begin by reviewing the priority ordering on reasons set out in Horty (2019), which is simply the most straightforward transcription to the dimensional setting of the ordering originally defined (Horty 2011; Horty and Bench-Capon 2012) only in the standard setting. For motivation, let us return to the case  $c_1 = \langle X_1, r_1, s_1 \rangle$ —where  $X_1 = \{\langle d_1, 30 \rangle, \langle d_2, 60 \rangle\}$ , where  $r_1 = \{M_{d_1,12}^\delta\} \rightarrow \delta$ , and where  $s_1 = \delta$ —and ask what information is carried by this case; what is the court telling us with its decision? Most explicitly, with its decision for the defendant on the basis of the rule  $r_1$ , the court is telling us that the reason for its decision—that is,  $Premise(r_1)$ , the premise of the rule—carries more weight, or has higher priority, than any reason for the plaintiff that holds in  $X_1$ , the fact situation of the case. But if  $Premise(r_1)$  itself has higher priority than any reason for the plaintiff that holds in  $X_1$ , the court must also be telling us, at least implicitly, that any other reason for the defendant that is at least as strong as  $Premise(r_1)$  must likewise have higher priority than any reason for the plaintiff that holds in this situation.

To capture this idea formally, we first recall that a reason  $Z$  for the defendant is at least as strong as  $Premise(r_1)$  whenever  $Z \Vdash Premise(r_1)$ , and that a reason  $W$  for the plaintiff holds in the situation  $X_1$  whenever  $X_1 \models W$ . If we let  $<_{c_1}$  represent the priority relation on reasons derived from the particular case  $c_1$ , then, the force of the court's decision in this case is that: where  $W$  is a reason favoring the plaintiff and  $Z$  is a reason favoring the defendant, then  $W <_{c_1} Z$  just in case  $Z \Vdash Premise(r_1)$  and  $X_1 \models W$ . Generalizing, we arrive at the following definition:

**Definition 4 (Priority ordering derived from a case)** Let  $c = \langle X, r, s \rangle$  be a case, and let  $W$  and  $Z$  be reasons favoring the sides  $\bar{s}$  and  $s$  respectively. Then the relation  $<_c$  representing the priority ordering on reasons derived from the case  $c$  is defined by stipulating that  $W <_c Z$  if and only if (1)  $Z \Vdash Premise(r)$  and (2)  $X \models W$ .

This definition can be illustrated, in the case of our example, by considering  $\{M_{d_1,60}^\pi\}$ , a reason for the plaintiff based on the fact that the defendant spent no more than five years, or sixty months, abroad. How does this reason compare to  $Premise(r_1)$ , the reason  $\{M_{d_1,12}^\delta\}$  for the defendant? Well, we have  $Premise(r_1) \Vdash Premise(r_1)$ , of course, and also  $X_1 \models \{M_{d_1,60}^\pi\}$ , since  $\{M_{d_1,60}^\pi\}$  holds in the situation  $X_1$ . It follows from our definition, therefore, that  $\{M_{d_1,60}^\pi\} <_{c_1} Premise(r_1)$ —that is, the court's decision in the case  $c_1$  implies that staying abroad for a year or longer is a stronger reason in favor of change of domicile than staying abroad for five years or less is against change of domicile.

Once we have defined the priority ordering on reasons derived from a single case, we can lift this idea to an ordering  $<_\Gamma$  derived from an entire case base  $\Gamma$  by stipulating

that one reason has a higher priority than another according to the case base whenever that priority relation is supported by some case from the case base:

**Definition 5 (Priority ordering derived from a case base)** Let  $\Gamma$  be a case base, and let  $W$  and  $Z$  be reasons. Then the relation  $<_{\Gamma}$  representing the priority ordering on reasons derived from  $\Gamma$  is defined by stipulating that  $W <_{\Gamma} Z$  if and only if  $W <_c Z$  for some case  $c$  from  $\Gamma$ .

And we can then define a case base as consistent as long as it does not provide conflicting information about the priority ordering among reasons—telling us, for some pair of reasons, that each has a higher priority than the other:

**Definition 6 (Consistent case bases)** Let  $\Gamma$  be a case base with  $<_{\Gamma}$  its derived priority ordering. Then  $\Gamma$  is inconsistent if and only if there are reasons  $W$  and  $Z$  such that  $W <_{\Gamma} Z$  and  $Z <_{\Gamma} W$ , and consistent otherwise.

Given this notion of consistency, the reason model of constraint applies, in the first instance, to the rules on the basis of which a court can reach its decision. Here, the intuition is that, in confronting a new situation against the background of an existing case base, the court is required to reach its decision on the basis of a rule that does not introduce inconsistency into that case base.

**Definition 7 (Reason model constraint on rule selection)** Let  $\Gamma$  be a case base and  $X$  a fact situation confronting the court. Then the reason model of constraint on rule selection requires the court to base its decision on some rule  $r$  supporting an outcome  $s$  such that the new case base  $\Gamma \cup \{\langle X, r, s \rangle\}$  is consistent.

But of course, once this constraint on rule selection is in place, the reason model can naturally be interpreted as requiring a decision for a particular side just in case every rule satisfying the constraint on rule selection supports that side.

**Definition 8 (Reason model constraint on decision)** Let  $\Gamma$  be a case base and  $X$  a fact situation confronting the court. Then the reason model of constraint on decision requires the court to reach a decision in  $X$  for the side  $s$  if and only if every rule satisfying the constraint on rule selection supports the side  $s$ .

To illustrate this notion of constraint, we consider two scenarios, each of which is based on the background case base  $\Gamma_1 = \{c_1\}$ , containing as its sole member the familiar  $c_1 = \langle X_1, r_1, s_1 \rangle$ , where  $X_1 = \{\langle d_1, 30 \rangle, \langle d_2, 60 \rangle\}$ , where  $r_1 = \{M_{d_1,12}^{\delta}\} \rightarrow \delta$ , and where  $s_1 = \delta$ . The fact situation from this case is depicted as a point in the coordinate space from Fig. 1, with the horizontal and vertical axes representing the dimensions  $d_1$  and  $d_2$ , length of time abroad and proportion of income earned abroad, and with values along each dimension more distant from the origin favoring the defendant more strongly; the diagram also indicates the range of situations in which  $Premise(r_1) = \{M_{d_1,12}^{\delta}\}$  holds, so that the rule of the case  $c_1$  applies.<sup>4</sup>

For our first scenario, imagine that a court confronts the new fact situation  $X_2 = \{\langle d_1, 36 \rangle, \langle d_2, 10 \rangle\}$ , also depicted in Fig. 1, representing a defendant who easily

<sup>4</sup> Related figures can be found Bench-Capon and Atkinson (2017a).





than the premise of the previous rule. In particular, as in the case  $c_2$ , the new court might consistently decide that  $Premise(r_1) <_{c_2} Premise(r_2)$ —that is, earning no more than twenty five percent of income abroad is a stronger reason against change of domicile than spending at least a year abroad is in favor of change of domicile.

For our second scenario, still working against the background of the case base  $\Gamma_1 = \{c_1\}$ , consider the new fact situation  $X_3 = \{\langle d_1, 15 \rangle, \langle d_2, 65 \rangle\}$ , again depicted in Fig. 1, representing a defendant who spent fifteen months abroad while earning sixty-five percent of his or her income abroad. Imagine that this situation comes before a court that evaluates change of domicile cases against very high standards for proportion of income earned abroad—suppose, in fact, that the court feels that change of domicile requires earning more than seventy-five percent of income abroad. A court like this would prefer to rule in favor of the plaintiff in the situation  $X_3$  on the grounds that the defendant fails the seventy-five percent test. This decision would be represented by the case  $c_3 = \langle X_3, r_3, s_3 \rangle$ , where  $X_3$  is as above, where  $r_3 = \{M_{d_2,75}^\pi\} \rightarrow \pi$ , and where  $s_3 = \pi$ .

Again we ask whether the court can rule as it prefers, and the answer this time is that it cannot, since the resulting case base  $\Gamma_1 \cup \{c_3\}$  is inconsistent. To see this, we note that the pair of cases  $c_1$  and  $c_3$  belonging to this case base would generate an inconsistent ordering on the premises of its case rules, since each of these rules would then hold in the fact situation of the other case, where the other rule was preferred. More exactly, we have  $Premise(r_1) \vdash \neg Premise(r_1)$ , of course, and also  $X_1 \models Premise(r_3)$ , from which it follows that  $Premise(r_3) <_{c_1} Premise(r_1)$ ; likewise we have  $Premise(r_3) \vdash \neg Premise(r_3)$  and also  $X_3 \models Premise(r_1)$ , from which it follows that  $Premise(r_1) <_{c_3} Premise(r_3)$ . What this scenario illustrates is that it is not possible to find for the plaintiff in the situation  $X_3$  on the basis of proportion of income earned abroad, since any reason favoring the plaintiff on the basis of proportion of income that holds in  $X_3$  would hold in  $X_1$  as well.

#### 4 Modifying the reason model

So far, so good—but now consider a third scenario. Imagine that the situation  $X_3$  comes before a court that cares little about proportion of income earned abroad, but applies stricter standards than the  $c_1$  court for length of time abroad. The new court would prefer to rule against change of domicile in this situation, and so for the plaintiff, on the grounds that the defendant failed to spend more than two years abroad—that is, that the actual period abroad favors the plaintiff at least as much as a period of two years, or twenty-four months. The resulting decision would be represented by the new case  $c_4 = \langle X_4, r_4, s_4 \rangle$ , where  $X_4 = X_3$ , where  $r_4 = \{M_{d_1,24}^\pi\} \rightarrow \pi$ , and where  $s_4 = \pi$ .

Can the new court rule as it prefers? It can, according to the reason model, since the resulting case base  $\Gamma_1 \cup \{c_4\}$  is consistent.<sup>6</sup> Of course, there is, quite plainly, a disagreement of sorts between the  $c_1$  and  $c_4$  courts, since the  $c_1$  court bases its decision on the rule that any period of a year or longer abroad is sufficient to justify change of

<sup>6</sup> Again, the proof of consistency is similar to that from Example 1 of Horty (2019).

fiscal domicile, while the  $c_4$  court relies on the rule that there is no change of domicile as long as the period abroad is two years or less. But according to the reason model, a disagreement like this does not rise to the level of inconsistency.

What this scenario illustrates is the general result that, as long as there is some dimension on which a new fact situation favors the winning side of a previous case less strongly than the fact situation from that case itself, then, no matter what the rule of the previous case, it will be consistent with that case to decide the new fact situation for the opposite side.<sup>7</sup> It is exactly this result that leads, in the dimensional setting, to the collapse of the reason model into the result model, and so to the feeling that the reason model should be modified.

In order to motivate my proposed modification, I want to return to the rationale for the original reason model, and especially to its justification for the conclusion that the case base  $\Gamma_1 \cup \{c_4\}$ , considered just above, is consistent. How could anyone think that this case base is consistent? How could anyone think, in light of the earlier  $c_1$  decision for change of domicile on the grounds that the defendant spent at least a year abroad, that a later court could consistently decide in  $c_4$  against change of domicile on the grounds that the defendant in that case, who did in fact spend at least a year abroad, failed to stay abroad for two years.

Well, to understand how this  $c_4$  decision could be regarded as consistent, on the reason model, let us recall our first scenario, where, in light of the same  $c_1$  decision for change of domicile on the grounds that the defendant spent at least a year abroad, a later court decided against change of domicile in  $c_2$  on the grounds that the defendant in that case, who had spent at least a year abroad, failed to earn at least twenty-five percent of income abroad. In that scenario, we concluded that the  $c_2$  decision was consistent with the earlier  $c_1$  decision because the new situation  $X_2$  presented new reasons for the plaintiff—in particular, the reason  $Premise(r_2) = \{M_{d_2,25}^\pi\}$ —that did not hold in the earlier situation  $X_1$ . Because these new reasons failed to hold in the earlier situation, they were, therefore, not ordered relative to the reason  $Premise(r_1) = \{M_{d_1,12}^\delta\}$ , justifying the earlier decision, and so could consistently be assigned a higher priority than this earlier reason.

With this argument for consistency in mind, we can return to the current scenario, where, as it turns out, exactly the same argument can be deployed to support the consistency of the  $c_4$  decision. To spell it out: the new situation  $X_4$  presents new reasons for the plaintiff—in particular,  $Premise(r_4) = \{M_{d_1,24}^\pi\}$ —that likewise do not hold in the earlier situation  $X_1$ , that were therefore not ordered relative to the reason  $Premise(r_1) = \{M_{d_1,12}^\delta\}$ , justifying the earlier decision, and so can likewise consistently be assigned a higher priority than this earlier reason.

Since the reason model argument for consistency in  $c_2$  applies equally well in  $c_4$ , it follows that, if we want to allow consistency in the first of these two cases but not the second, we will have to emend the reason model to reflect the difference between them. But what could the relevant difference be? Given that  $c_1$  had been justified with the reason  $Premise(r_1) = \{M_{d_1,12}^\delta\}$ , and neither  $Premise(r_2) = \{M_{d_2,25}^\pi\}$  nor  $Premise(r_4) = \{M_{d_1,24}^\pi\}$  held in the earlier situation, what basis could we have for later allowing  $Premise(r_2)$  but not  $Premise(r_4)$ , to be consistently assigned a

<sup>7</sup> This result is verified as Observation 3 of Horty (2019), and then discussed in Example 5.

higher priority than the original  $Premise(r_1)$ ? The proposal set out here is based on the idea that the framework of dimensions allows us to define new priority relations among reasons according to which, as a result of the  $c_1$  decision, the reason  $Premise(r_1) = \{M_{d_1,12}^\delta\}$  must be assigned a priority higher than that of  $Premise(r_4) = \{M_{d_1,24}^\pi\}$ , but not higher than that of  $Premise(r_2) = \{M_{d_2,25}^\pi\}$ , so that, in later cases,  $Premise(r_2) = \{M_{d_2,25}^\pi\}$  can consistently be ranked above  $Premise(r_1) = \{M_{d_1,12}^\delta\}$  but  $Premise(r_4) = \{M_{d_1,24}^\pi\}$  cannot.

How can we define these new priority relations? The definition I propose moves through two steps. The first is entirely notational: where  $V$  is a reason favoring the side  $s$ , we let

$$\bar{V} = \{M_{d,p}^{\bar{s}} : M_{d,p}^s \in V\}$$

be a reason favoring the side  $\bar{s}$ , built from magnitude factors addressing the same dimensions as those in  $V$  and using the same reference values, but favoring the opposite side. To illustrate: where  $Premise(r_1) = \{M_{d_1,12}^\delta\}$  is a reason favoring the defendant on the grounds that he or she spent a year or more abroad,  $\overline{Premise(r_1)} = \{M_{d_1,12}^\pi\}$  is a reason favoring the plaintiff on the grounds that he or she spent a year or less abroad. Next, and more substantially, where  $c = \langle X, r, s \rangle$  is a case decided for the side  $s$  with the reason  $Premise(r)$  as justification, and  $W$  is a reason favoring the opposite side  $\bar{s}$ , we stipulate that  $\overline{Premise(r)}$  is to be assigned a higher priority than  $W$  on the basis of  $c$  whenever  $\overline{Premise(r)} \Vdash W$ .

The intuition behind this definition is this: By appealing  $Premise(r)$  to justify its decision for  $s$ , the court is implicitly asserting that  $\overline{Premise(r)}$  is a stronger reason for  $s$  than  $\overline{Premise(r)}$  is for  $\bar{s}$ . And then of course, as we have seen, if  $\overline{Premise(r)} \Vdash W$ , we know that  $\overline{Premise(r)}$  is a stronger reason than  $W$  is for  $\bar{s}$ , from which it follows—given that  $Premise(r)$  is a stronger reason for  $s$  than  $\overline{Premise(r)}$  is for  $\bar{s}$ —that  $\overline{Premise(r)}$  must likewise be stronger, or have higher priority, as a reason for  $s$  than  $\overline{Premise(r)}$  does for  $\bar{s}$ .

Returning to our third scenario for illustration: By appealing to  $Premise(r_1) = \{M_{d_1,12}^\delta\}$  to justify a judgment for the defendant, the  $c_1$  court is implicitly asserting that  $\overline{Premise(r_1)} = \{M_{d_1,12}^\pi\}$  is a stronger reason for the defendant than  $\overline{Premise(r_1)} = \{M_{d_1,12}^\pi\}$  is for the plaintiff—spending a year or more abroad is a stronger reason for the defendant than spending a year or less abroad is for the plaintiff. And then since  $\overline{Premise(r_1)} \Vdash \{M_{d_1,24}^\pi\}$ , we know that  $\overline{Premise(r_1)} = \{M_{d_1,12}^\pi\}$  is a stronger reason than  $Premise(r_4) = \{M_{d_1,24}^\pi\}$  for the plaintiff, so that  $Premise(r_1) = \{M_{d_1,12}^\delta\}$  must likewise be stronger, or have higher priority, as a reason for the defendant than  $Premise(r_4) = \{M_{d_1,24}^\pi\}$  does for the plaintiff—spending a year or more abroad must have higher priority as a reason for the defendant than spending two years or less abroad does for the plaintiff.

The new priority relations definable on the basis of dimensional information can be incorporated very simply into our previous definition of the priority ordering among reasons derived from a case:

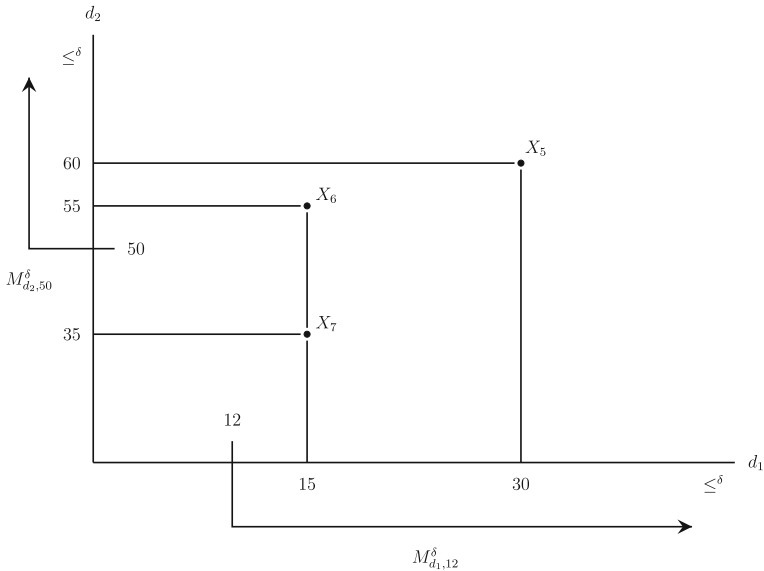


Fig. 2  $r_5 = \{M_{d_1,12}^\delta, M_{d_2,50}^\delta\} \rightarrow \delta$

**Definition 9 (Priority ordering derived from a case: modified)** Let  $c = \langle X, r, s \rangle$  be a case, and let  $W$  and  $Z$  be reasons favoring the sides  $\bar{s}$  and  $s$  respectively. Then the relation  $<_c$  representing the priority ordering on reasons derived from the case  $c$  is defined by stipulating that  $W <_c Z$  if and only if (1)  $Z \Vdash \text{Premise}(r)$  and either (2a)  $X \models W$  or (2b)  $\overline{\text{Premise}(r)} \Vdash W$ .

The modified reason model then results simply by replacing the previous Definition 4 with this new Definition 9, leaving everything else unchanged.

It is easy to see that this modified reason model agrees with the original in our first and second scenarios. In our third scenario, where the original model yields the troubling result that the  $c_4$  decision is consistent with  $\Gamma_1 = \{c_1\}$ , the modified reason model yields the more natural result that this decision is inconsistent. More exactly, we have  $\text{Premise}(r_1) <_{c_4} \text{Premise}(r_4)$ , since (1)  $\text{Premise}(r_4) \Vdash \text{Premise}(r_4)$  and (2a)  $X_4 \models \text{Premise}(r_1)$  holds; but we also have  $\text{Premise}(r_4) <_{c_1} \text{Premise}(r_1)$ , since (1)  $\text{Premise}(r_1) \Vdash \text{Premise}(r_1)$  and—although (2a)  $X_1 \models \text{Premise}(r_4)$  fails—we now have (2b)  $\overline{\text{Premise}(r_1)} \Vdash \text{Premise}(r_4)$ .

In order to see how the modified rule model works with reasons addressing multiple dimensions, it will be useful to consider two additional scenarios. Suppose a court faced once more with the familiar fact situation  $X_1 = \{(d_1, 30), (d_2, 60)\}$  again decides for the defendant, but this time on the basis of the more restrictive consideration that the defendant both spent at least a year abroad and earned at least fifty percent of income abroad. This decision can be represented through the new case  $c_5 = \langle X_5, r_5, s_5 \rangle$ , where  $X_5 = X_1$ , where  $r_5 = \{M_{d_1,12}^\delta, M_{d_2,50}^\delta\} \rightarrow \delta$ , and where  $s_5 = \delta$ . The fact situation and rule from this case are depicted in Fig. 2.

Against the background of the case base  $\Gamma_2 = \{c_5\}$  containing this case, consider the new fact situations  $X_6 = \{\langle d_1, 15 \rangle, \langle d_2, 55 \rangle\}$  and  $X_7 = \{\langle d_1, 15 \rangle, \langle d_2, 35 \rangle\}$ , also depicted in Fig. 2. Suppose the court wishes to decide  $X_6$  for the plaintiff on the grounds that the defendant failed to spend at least two years abroad, a decision that would be represented through the case  $c_6 = \langle X_6, r_6, s_6 \rangle$ , where  $X_6$  is as above, where  $r_6 = \{M_{d_1,24}^\pi\} \rightarrow \pi$ , and where  $s_6 = \pi$ . According to the modified version of the rule model, though not the original, this decision would be inconsistent with  $\Gamma_2$ . We have  $\underline{Premise(r_6)} <_{c_5} \underline{Premise(r_5)}$ , since (1)  $\underline{Premise(r_5)} \vdash \underline{Premise(r_5)}$  and (2b)  $\underline{Premise(r_5)} \vdash \underline{Premise(r_6)}$ . But we also have  $\underline{Premise(r_5)} <_{c_6} \underline{Premise(r_6)}$ , since (1)  $\underline{Premise(r_6)} \vdash \underline{Premise(r_6)}$  and (2a)  $X_6 \models \underline{Premise(r_5)}$ .

On the other hand, suppose the court wishes to decide  $X_7$  for the plaintiff for exactly the same reason, because the defendant failed to spend at least two years abroad, leading to  $c_7 = \langle X_7, r_7, s_7 \rangle$ , where  $X_7$  is as above, where  $r_7 = \{M_{d_1,24}^\pi\} \rightarrow \pi$ , and where  $s_7 = \pi$ . This time the decision would be consistent with  $\Gamma_2$ . We again have  $\underline{Premise(r_7)} <_{c_5} \underline{Premise(r_5)}$ , since (1)  $\underline{Premise(r_5)} \vdash \underline{Premise(r_5)}$  and (db)  $\underline{Premise(r_5)} \vdash \underline{Premise(r_7)}$ . But we do not have  $\underline{Premise(r_5)} <_{c_7} \underline{Premise(r_7)}$ , since although (1)  $\underline{Premise(r_7)} \vdash \underline{Premise(r_7)}$  holds, of course, both (2a)  $X_7 \models \underline{Premise(r_5)}$  and (2b)  $\underline{Premise(r_7)} \vdash \underline{Premise(r_5)}$  fail.

## 5 Conclusion

The point of this note was to propose a modification to the reason model of constraint for application in the dimensional setting. The proposed modification separates the reason model from the result model and yields more attractive outcomes in certain situations, such as those explored in our third scenario.

This proposal should, of course, be compared with those of Bench-Capon and Atkinson (2017a, b, 2018) and Rigoni (2018). Unfortunately, those proposals are developed within frameworks sufficiently different from my own (and from each other), that any detailed comparison will have to wait. However, a few words are in order:

The chief expressive difference between the current account and Rigoni’s is that Rigoni limits consideration to a single dimension, while the current treatment allows multiple dimensions—indeed, it is the treatment of multiple dimensions that has proved most challenging. This difference raises two natural questions: first, does the current account agree with Rigoni’s when restricted to a single dimension, and second, can Rigoni’s method be generalized to allow multiple dimensions? In addition to this expressive difference, there are a number of representational, or conceptual differences. For example, rather than reifying dimensions, as I do, and then evaluating reasons at points in multi-dimensional space, Rigoni constructs dimensions as sets of reasons; this allows him to explore interesting questions about the ways in which dimensions might be established from the cases in a case base, and also about dimensions that are, in one way or another, incomplete. Another important difference is that Rigoni’s account relies on what he calls the “switching point”—a point, or region, along a certain dimension at which factors cease to favor one side and begin favoring

the other.<sup>8</sup> I deny that there is a switching point, supposing instead that reasons favoring each side of a dispute can be found all along any given dimension, varying only in strength.

Bench-Capon and Atkinson's ideas are developed using techniques of abstract dialectical frameworks (Brewka and Woltran 2010), applied to legal reasoning in their previous work with Al-Abdulkarim (Al-Abdulkarim et al. 2016). In contrast to the current approach, and to Rigoni's, Bench-Capon and Atkinson choose not to appeal to factors, or to reasons, in analyzing reasoning with dimensions. They recognize that such appeal is convenient for a "conventional style" of argument "based on propositions and rules," but reject this approach on the grounds that it would exclude the proper treatment of tradeoffs and balance between dimensions; instead, they pursue a "different flavor of argumentation, based on geometry rather than rules."<sup>9</sup> Although this promising work is still at very preliminary stage, it seems fair to ask, even at this early point, exactly why Bench-Capon and Atkinson feel that the conventional approach based on rules and propositions excludes a proper treatment of tradeoffs among dimensions. Within the current framework, there seems to be no reason why a pattern of decisions might not reveal, for example, that the courts allow a year abroad to justify change of domicile for defendants who have earned fifty percent of income abroad, but require five years abroad to justify change of domicile for defendants who have earned only twenty-five percent of income abroad.

Both Bench-Capon and Atkinson and Rigoni argue that an adequate representational framework must include ordinary factors as well as dimensions. I agree, and in my previous work have defined two different mappings of standard information, including ordinary factors, into the dimensional setting, and explored conditions under which the reason model of constraint carries over, on the basis of these mappings, from the standard to the dimensional setting.<sup>10</sup> Whether these mappings continue to preserve the modified reason model developed here is an open question, but in any case, I do feel that the reason model should be unified, applying to ordinary factors and dimensional factors at once.

Finally, putting aside comparison to previous work, it is important to note that the modified reason model proposed here raises interesting conceptual questions of its own. Consider, one last time, the case  $c_1 = \langle X_1, r_1, s_1 \rangle$ —where  $X_1 = \{\langle d_1, 30 \rangle, \langle d_2, 60 \rangle\}$ , where  $r_1 = \{M_{d_1,12}^\delta\} \rightarrow \delta$ , and where  $s_1 = \delta$ —in which the court confronting a defendant who had spent two and a half years abroad found for change of domicile on the grounds that the defendant had stayed abroad for at least a year. Now imagine that a new court confronting the new situation  $X_8 = \{\langle d_1, 6 \rangle, \langle d_2, 60 \rangle\}$ , in which a defendant has spent a mere six months abroad, rules against change of domicile, and so for the plaintiff, on the grounds that the defendant had stayed abroad for less than two years. This decision would be represented by the case  $c_8 = \langle X_8, r_8, s_8 \rangle$ , where  $X_8$  is as above, where  $r_8 = \{M_{d_1,24}^\pi\} \rightarrow \pi$ , and where  $s_8 = \pi$ . According to the modified reason model proposed here, but not according to the original, these two cases would be inconsistent—so that a court could not, for example, consistently

<sup>8</sup> See Sect. 3 of Rigoni (2018).

<sup>9</sup> See Sect. 2 of Bench-Capon and Atkinson (2017b).

<sup>10</sup> See Sect. 4 of Horty (2017) and Appendix A of Horty (2019).

reach the  $c_8$  decision against the background of a case base containing  $c_1$ . It is striking, however, that neither of the rules from these two cases applies to the fact situation from the other—that is,  $r_1$  does not apply to  $X_8$  and  $r_8$  does not apply to  $X_1$ . Any inconsistency must therefore result entirely from structural relations between the case rules themselves, contrary to what many feel is the common law idea that inconsistency is not an abstract feature of rule systems, but arises only from the application of rules to particular, concrete situations.

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