A Contrario Argument and Default Reasoning

John Horty

*** DRAFT ***

Version of: August 15, 2021

Overview

I have to admit that, prior to reading Duarte d'Almeida's paper, I was unaware of the controversy involving argument *a contrario*, or *expressio unius*.[1] But I enjoyed learning about it, and about the work surrounding this issue in the literature on legal interpretation and informal argumentation. I admire Duarte d'Almeida's carefully documented overview of the problem. I especially admire his criticisms of the standard account and his own very sophisticated positive proposal. Part of what I like most about the paper is that it does not begin with a high-level theory of the nature of law, deduce an account of what legal reasoning must be from that, and then force the specific phenomena under investigation to fit that account. Instead, it arrives at a more general proposal on the basis of careful and relatively theory-neutral reflection on the phenomena itself. It is bottom-up work, rather than top-down.

In this note I will first summarize the problem presented by *a contrario* reasoning, then what Duarte d'Almeida calls the standard response to this problem and his criticism of this standard response. It is impossible, in a brief note, to summarize Duarte d'Almeida's positive proposal, which is, in any case, elegantly presented in his paper. I will, however, highlight some of its most important features. Next, I will sketch how I would approach the problem of *a contrario* reasoning, an approach that I suspect would be shared by many other researchers in the field of AI and Law, and in logical AI more generally. I cannot compare this approach to Duarte d'Almeida's here, but I will argue that the two approaches share some important features, which suggests that a detailed comparison would be useful.

Duarte d'Almeida on *a contrario* reasoning

I begin with a passage from Justice Scalia, quoted by Duarte d'Almeida, which will serve to introduce the form of argument under consideration. Speaking of the cannons of legal

[1] Duarte d'Almeida (2021).

interpretation, which, he says, have been "widely criticized, indeed even mocked," by modern legal commentators, Scalia writes:

> Many of the canons were originally in Latin, and I suppose that alone is enough to render them contemptible. One for example, is *expressio unius es exclusio alterius*. Expression of the one is exclusion of the other. What it means is this: If you see a sign that says children under twelve may enter free, you should have no need to ask whether your thirteen-year-old must pay. The inclusion of the one class is an implicit exclusion of the other .... All of this is so commonsensical that, were the canons not couched in Latin, you would find it hard to believe anyone could criticize them.[2]

Suppose, then, that you are taking your thirteen-year-old, Chris, to the theater, and you see a sign saying

(1) Children under twelve may enter for free.

According to Scalia, you should be able to conclude at once—by *a contrario* or *expressio unius* reasoning, because inclusion of the one is implicit exclusion of the other—that Chris cannot enter for free. What Scalia thinks, in other words, is that (1) and

(2) Chris is not under twelve

immediately support the conclusion

(3) Chris cannot enter for free.

The problem, though, is that the inference from (1) and (2) to (3) is not logically valid. Of course, logic would yield (3) as a conclusion if we could somehow establish

(4) Children who are not under twelve cannot enter for free,

[2] Scalia (1997, pp. 25–26).

since (4) and (2) imply (3). But the inference from (1) to (4) is not logically valid either. Indeed, these inferences—from (1) and (2) to (3), or from (1) to (4)—are exactly the kind of inferences that introductory logic instructors around the world struggle to teach their students to avoid.

So how do we solve this problem—how do we support Scalia's very reasonable intuition that Chris cannot enter for free, even though the inference that seems to support this conclusion is invalid? According to Duarte d'Almeida, the standard solution to this problem is to suppose that, although (1) does not imply (4) as a matter of logic, there are many situations in which a statement of the form (1) can naturally be understood to include the corresponding statement of the form (4) as well. To put the matter a bit more formally, let us take $\mathit{Twelve}(x)$ to mean that x is under twelve and $\mathit{Free}(x)$ to mean that x can enter for free. Then according to the standard solution, although what (1) literally says can be represented as (1′), there are supposed to be many situations in which (1) can naturally be interpreted as expressing (1″) instead:

(1′) For all x: if $\mathit{Twelve}(x)$, then $\mathit{Free}(x)$,

(1″) For all x: $\mathit{Twelve}(x)$ if and only if $\mathit{Free}(x)$.

And it is in these situations, according to the standard solution, that *a contrario* reasoning from (1) and (2) to (3) is valid.

How do we characterize situations in which statements of the form (1) can be interpreted to mean (1″), rather than (1′)? This is a question that need not concern us—perhaps the answer lies in an appeal to Gricean considerations, perhaps an appeal to legislator intentions, or in our example, the intentions of the theater. What is important, for our purposes, is not how to characterize situations in which statements of the form (1) can be interpreted to mean that (1″), but simply that there are such situations and that, according to the standard solution, these are the situations in which *a contrario* reasoning is legitimate.

Duarte d'Almeida offers, throughout his paper, a number of criticisms of this standard solution. All are interesting and persuasive, but one is absolutely conclusive. Here it is. Suppose, as Scalia thinks, *a contrario* reasoning is supported in the theater situation. According to the standard account, this must be because the sign containing the statement (1) is interpreted to mean that (1″). Even though what it actually says is that children under twelve can enter for free, it must be interpreted as meaning, in addition, that only children under twelve can enter for free. Now imagine that, after putting out this first sign, the theater later puts out another sign saying

(5) Retirees may enter for free.

In that case, as long as the domain of discourse contains some retiree older than twelve, the theater has contradicted itself. Why? Well, suppose Lynn is such a retiree. As we have seen, it follows from (1), interpreted as (1″), that Lynn cannot enter for free, but it now follows from (5) that Lynn can enter for free. This cannot be right. Surely the theater can first offer free admission to children under twelve, and then generously extend the benefit to retirees, without running afoul of logic.

Motivated in part by the failure of this standard solution, and in part by broader reflections on the nature of legal reasoning developed in his previous work, Duarte d'Almeida advances an alternative proposal to account for the acceptability of *a contrario* reasoning, in those cases in which it is acceptable. Although I cannot summarize this proposal here, I do want to highlight some of its important features.

First, according to Duarte d'Almeida, there is, in each case, a default decision. "The court needs to know what the 'default' decision, as we might call it, should be".... this correct default decision "is also a matter of law" (Section 4.1).[3]

Second, with regard to any particular question—such as whether or not they can enter the theater for free—individuals have an existing, or default, "normative status." Typically,

[3]Section numbers refer to Duarte d'Almeida (2021).

the default decision in a case will be to "decline to alter [an individual']s existing normative position, or status (Section 4.1).

Third, the role of a rule or provision—such as the rule that children under twelve can enter for free—is typically to "introduce some change in the normative status of those to whom it is meant to apply" (Section 5.3).

Fourth, a conclusion of *a contrario* reasoning—such as the conclusion that Chris cannot enter for free—is typically not something that actually follows from the relevant rule or provision. Instead, it reflects the fact that the provision "does not regulate" the normative status of some individual, such as Chris, so that that individual's default "normative status remains unchanged" (Section 5.3).

Fifth, legal rules should not be "understood—as the standard view understands them—as universal conditionals," so that, for example, "the words of the sign [at the theater] do not themselves translate into the formulation of a rule, understood as a universal condition" (Section 5.1).

Sixth, because legal rules do not express universal conditionals, legal reasoning does not proceed through the "legal syllogism," according to which applying the law involves simply subsuming an individual under some general principle, expressed as a universal conditional, which dictates a conclusion (Section 4.2).

Seventh, instead of simply moving through the legal syllogism, legal reasoning relies in important ways on "second-order" considerations about "a provision's applicability (or . . . inapplicability)" to particular individuals, such as the inapplicability of the provision about children under twelve in the case of Chris (Section 4.2).

Default reasoning: a first pass

Let us return to Scalia's observation: "If you see a sign that says children under twelve may enter free, you should have no need to ask whether your thirteen-year-old must pay." The problem, recall, is that the inference featured in this observation was not valid in logic. But we can now ask: Which logic? To be sure, the inference is not valid in classical logic, originating in the 19th Century with the work of Frege, and designed to formalize mathematical reasoning. But Scalia does not say that the inference is valid in classical logic. What he says is that it is "commonsensical"—and as it turns out, there are also a number of logics, originating in computer science, widely applied in the subfield of AI and Law and elsewhere, designed with the explicit aim of formalizing aspects of commonsense reasoning.

In fact, many of the problems that first motivated these new logics are strikingly similar to those presented by *a contrario* reasoning. Consider, for example, an airline flight database, which we can think of as a series of statements of the form $\mathit{Flight}(N, C_1, C_2)$, indicating that flight number N flies from city C_1 to city C_2.[4] Imagine that the flight database for a (rather small) hypothetical airline contains only the following three statements:

$$\mathit{Flight}(196, \mathit{LosAngeles}, \mathit{Miami}),$$
$$\mathit{Flight}(352, \mathit{Miami}, \mathit{NewYork}),$$
$$\mathit{Flight}(467, \mathit{NewYork}, \mathit{Paris}).$$

From this database, we can draw a number of conclusions using ordinary classical logic. We can conclude, for example, that the airline connects Miami to Paris, in the sense that there is a city such that, on the airline, you can fly from Miami to that city, and then from that city to Paris. In a similar way, classical logic allows us to conclude that the airline connects Los Angeles to Paris.

But now, suppose you ask: Does the airline fly from Lisbon to Edinburgh? Of course the answer is No. A ticket agent, after a glance at the database, would tell you No. Google Flights or TripAdvisor would tell you No. But this conclusion does not follow from logic. Classical logic simply does not allow you to conclude, from the set of statements listed

[4]Our simple database language ignores, of course, time of departure and arrival, aircraft type, seats, and all the other fields that would be found in records from an actual flight database.

above, that there is no flight from Lisbon to Edinburgh.[5] The inference from the database to the conclusion is not valid in classical logic—it follows, instead, by a form of commonsense reasoning. How does our commonsense reasoning work, in this case? Very roughly, we are to assume, as a default, that the airline does not offer a flight from any particular city to any other—unless we have information to the contrary, we are to assume, for example, that there is no flight from Topeka to Riga. The database can then be thought of as a list of exceptions to this general default: cities that do happen to have flights between them. If there is no flight between cities explicitly listed in the database, as an exception to the general default, we are to suppose that the default assumption holds, and conclude that there is no such flight—from Topeka to Riga, or from Lisbon to Edinburgh. It is just as Scalia says. Inclusion of the one is exclusion of the other: from the fact that various flights are listed in the database, but a flight from Lisbon to Edinburgh is not, we are to conclude that there is no such flight.

In order to make formal sense of this kind of commonsense reasoning, and also to serve as a guide for implementations, a number of specialized logics were developed within the limited setting of deductive databases and logic programming languages: logics for "closed world reasoning," or for "predicate completion," or for "negation as failure."[6] In time, these specialized logics were subsumed within more general logics of commonsense and nonmonotonic reasoning. These more general logics have, I believe, a good deal philosophical relevance; and

[5]One way to see this is to note that, if logic actually did allow you to derive the conclusion that there is no flight from Lisbon to Edinburgh from the database of existing flights—if the absence of such a flight were somehow a logical consequence of the database—then it would have to follow as a matter of logic that the airline could never add such a flight. But that is absurd: the airline can easily add such a flight at any time.

[6]See, for example, Clark (1978) and Reiter (1978). Clark's idea of negation as failure is particularly relevant, since it allow us to move, from the premise that we cannot establish some proposition, to the conclusion that that proposition is false—so, for instance, from the fact that the database list no flight from Lisbon to Edinburgh, to the conclusion that there is no such flight, or from the premise that we cannot establish that Chris can enter for free, to the conclusion that he cannot enter for free.

in earlier work, I tried to establish the philosophical relevance of one of these more general logics—default logic—by showing in some detail how it could be used to formalize important patterns of normative and epistemic reasoning.[7] With some trepidation, I will now sketch how the kind *a contrario* reasoning at work in Scalia's example can likewise be captured in a very simple version of default logic.

Here is how our simple default logic works. There is ordinary, or hard, information, and there is default, or soft, information. We are to reason with the hard information in the usual way, on the basis of ordinary logic, but in addition, we are also supposed to accept all of the default information that is consistent with the hard information, as well as with whatever other default information we have accepted.[8]

So that is, basically, the logic. Now, returning to the theater example, let us suppose that there are three individuals—a, b, and c—that a and b are under twelve years old, and that c is not. Our hard information, then, includes the basic facts

$$Twelve(a),$$
$$Twelve(b),$$
$$\neg Twelve(c).$$

And let us suppose to begin with that the rule on the sign, interpreted literally as (1′), displayed earlier, is likewise included with the hard information:

$$(1') \quad \text{For all } x: \text{ if } Twelve(x), \text{ then } Free(x).$$

In addition to this hard information, our reasoning is also guided by a body of soft information according to which it is to be assumed, by default, that individuals cannot enter the

[7]See Horty (2012).

[8]In order to keep this presentation simple, we ignore complexities introduced by the fact that different sets of defaults might be separately consistent with the hard information though not mutually consistent with the hard information.

theater for free. This information is encoded in the three defaults

$$d_1 = \neg \mathit{Free}(a),$$
$$d_2 = \neg \mathit{Free}(b),$$
$$d_3 = \neg \mathit{Free}(c),$$

each of which represents the default assumption that a specific individual cannot enter for free.

What can we conclude from this information? Well, we reason with the hard information in the usual way, so applying ($1'$) to $\mathit{Twelve}(a)$ and $\mathit{Twelve}(b)$, we have $\mathit{Free}(a)$ and $\mathit{Free}(b)$—a and b can enter for free, since they are under twelve. We cannot accept the defaults d_1 or d_2, since these conflict with the hard information—we have just seen that a and b can enter for free. But d_3 is consistent with our existing set of conclusions, so we must accept this default as well. We thus conclude $\neg \mathit{Free}(c)$—the individual c cannot enter for free. If we suppose that c is Chris, the thirteen-year-old, we get Scalia's conclusion that Chris must pay to enter, even though the sign is given its literal reading ($1'$), rather than the biconditional reading ($1''$) thrust upon it by the standard account.

The account sketched here is similar, in many ways, to Duarte d'Almeida's proposal, as we can see by running through the features of this proposal listed a few pages back. First, obviously, the current account relies on defaults: the "default decision" for any particular individual is that that individual cannot enter the theater for free. The lack of permission to enter for free can be thought of as the existing "normative status" of an individual, so that, second, the default decision in the current account is simply the decision to "decline to alter" that existing normative status. Third, the role of a rule or provision, such as the sign saying that children under twelve can enter for free, is to "introduce some change in the normative status of those to whom it" applies, so that, as a result of this provision, a and b can enter for free. Fourth, in the current account, the *a contrario* conclusion that c cannot enter for free does not actually follow from the rule stated on the sign, but rather reflects the fact that the rule does not apply to c, so that this individual's default "normative status remains unchanged."

The current account also shares, in a sense, the seventh feature of Duarte d'Almeida's proposal: it relies on higher-order reasoning, since default conclusions are to be accepted only if they are consistent with the hard information, and reasoning about consistency is higher-order. But of course, this is not the kind of substantive higher-order reasoning that Duarte d'Almeida has in mind.

The current account does not, however, share the fifth and sixth features of Duarte d'Almeida's proposal. The rule stated on the sign is explicitly represented as a universal conditional—that is, as (1′). And, although the default reasoning concerning c moves in a different way, the application of the rule (1′) to the individuals to whom it does apply, a and b, proceeds directly through the legal syllogism.

Default reasoning: an elaboration

In fact, I think Duarte d'Almeida is right: legal rules are not universal conditionals. Really, it is hard to think of any useful normative generalizations that can be formulated as universal conditionals. My own view is that rules such as that appearing on the theater sign are best thought of as conditional defaults. What are these? Well, the simple defaults considered so far are just ordinary formulas of the form Y, which we are supposed to accept as long as they are consistent with available hard information and other accepted defaults. Conditional defaults, by contrast, are rules of the form $X \rightarrow Y$, which work as follows: if X is established, we are to accept Y as well, as long as Y is consistent with the hard information and other accepted default conclusions. A conditional default, then, functions just like a simple default once its premise is established, but is otherwise inert.

To illustrate, we return to our example and suppose that the rule from the sign is no longer represented as (1′)—this is, (1′) removed from the hard information. Instead, the rule is represented as a set of conditional defaults, according to which, for each individual, if that individual is under twelve, then, as a default, that individual can enter the theater for free:

$$d_4 = Twelve(a) \rightarrow Free(a),$$

$$d_5 = Twelve(b) \rightarrow Free(b),$$

$$d_6 = Twelve(c) \rightarrow Free(c).$$

We can ignore d_6, since the premise of this conditional default cannot be established: c is not under twelve. But the premises of d_4 and d_5 are established, belonging to our body of hard information. Should we, then, accept the conclusions of these defaults, $Free(a)$ and $Free(b)$—that a and b can enter for free?

Here, we run into a problem, since these conclusions conflict with the information provided by d_1 and d_2, which tell us that, as with any other individuals, the default assumption concerning a and b is that they cannot enter for free. The way we deal with this problem in default logic is by postulating a priority ordering among default rules, and assuming that, very roughly, if two defaults support conflicting conclusions, only the conclusion of the higher priority default is accepted.[9] Where do these priorities come from? There can be various sources. The law gives us principles such as *lex superiori* or *lex posteriori*, according to which rules issued by higher authorities, or issued later, are prioritized over rules issued by lower authorities, or issued earlier. The military gives us principles according to which orders issued by higher-ranking officers are prioritized over orders issued by lower-ranking officers, and direct orders are prioritized over standing orders. In some of the more sophisticated default logics, priorities among defaults are themselves established through the process of default reasoning.[10]

Returning to our example, let us suppose that, for whatever reason, the defaults d_4 and

[9]This statement of the policy for handling conflicts in prioritized default logic is sufficient for the examples considered in this note, but the problems presented by prioritized defaults can be complex, and are an important topic of contemporary research: for historical references, see Brewka (1994), Horty (2007), and Hansen (2008), and then Dung (2016) and Liao et al. (2019) for more recent work.

[10]The idea of reasoning about priorities among defaults within default logic itself was first suggested by Gordon (1993). The techniques were later refined by Brewka (1994) and, in particularly important work,

d_5 are prioritized over the previous d_1 and d_2—the assumption, based on the sign, that children under twelve can enter for free is prioritized over the background assumption that individuals in general cannot enter for free. In that case, we would accept the conclusions of these two new default rules, $\mathit{Free}(a)$ and $\mathit{Free}(b)$—again, a and b can enter for free. And just as before, since nothing interferes with d_3, we accept $\neg \mathit{Free}(c)$ as well—c cannot enter for free.

This new representation, then, yields the same results as our previous representation, where the rule on the sign was encoded as $(1')$, a universal conditional—both representations allow us to conclude that a and b can enter free, and, as desired, c cannot. But the new representation, with the rule cast in terms of defaults, is more flexible. Imagine that, in these pandemic times, the theater sets out another sign saying that no one with a fever can enter for free, and suppose it is established that b has a fever, so that our hard information is supplemented with a new fact:

$$\mathit{Fever}(b).$$

If the two rules are represented as universal conditionals—all children under twelve can enter for free, and no one with a fever can enter for free—then, since both rules apply to b, we would have a logical contradiction: b can enter for free on the basis of age, and b cannot enter for free on the basis of fever. But with a default representation, the new rule, like the previous rule, would be represented instead through a collection of defaults:

$$\begin{aligned} d_7 &= \mathit{Fever}(a) \to \neg \mathit{Free}(a), \\ d_8 &= \mathit{Fever}(b) \to \neg \mathit{Free}(b), \\ d_9 &= \mathit{Fever}(c) \to \neg \mathit{Free}(c). \end{aligned}$$

This yields more sensible results. Again, we can ignore d_7 and d_9, since it is not established that a or c has a fever. But b has a fever, and if we suppose, reasonably, that d_8 is prioritized

by Prakken and Sartor (1996; 1998); a discussion from a philosophical perspective is found in Chapter 5 of Horty (2012).

over d_5, since having a fever is more important than being under twelve, we will have to accept $\neg Free(b)$, the conclusion of this default.

Taken together, then, the information now presented to us—our hard information about age and fever, our default representation of the two rules—supports the following conclusions: $Free(a)$ on the basis of d_4, $\neg Free(b)$ on the basis of d_8, and $\neg Free(c)$ on the basis of d_3—a can enter for free on the basis of age, b cannot enter for free on the basis of fever, and c cannot enter on the basis of the default assumption that, in the absence of information to the contrary, people in general cannot enter for free. This seems like the right set of conclusions, under the circumstances.

Once our default account is elaborated to represent rules as conditional defaults, we can now see that this account shares all seven of the features noted earlier of Duarte d'Almeida's proposal. We have already considered the first four of these features. Moving on to the fifth, it is plain, of course, that legal rules are no longer represented as universal conditionals, but as defaults. And sixth, the application of these rules is no longer determined through the simple legal syllogism, but instead, seventh, through complex patterns of higher-order reasoning concerning, not only consistency, but also priority relations among conflicting defaults.

Conclusion

By focusing on their shared features, I have tried to highlight the similarities between Duarte d'Almeida's proposal and the account sketched here, grounded in default reasoning. But at least on the surface, the two approaches really are very different—Duarte d'Almeida's proposal does not involve defaults at all, but is organized around explicit higher-order reasoning about the classes of individuals to whom a rule is applicable. Given the similarities, as well as the differences, between these two approaches, a careful comparison would be very useful: Do the two approaches simply represent different ways of framing what might be, at a deeper level, the same idea, or are there more fundamental differences between them? It is, to my mind, a great virtue of Duarte d'Almeida's paper that it presents his proposal with enough clarity and precision that a comparison of this kind could actually be carried out.

I close with a conjecture, and then a suggestion. The controversy concerning *a contrario* argument is motivated by the fact that an attractive pattern of reasoning is not supported by classical logic—this is what Jansen, cited by Duarte d'Almeida, describes as its "logical validity problem."[11] As I have tried to show, however, this motivation may itself be suspect since, although it is not supported by classical logic, there are logics that do support this attractive pattern of reasoning, and do so in a natural way. My conjecture is that the misunderstanding concerning the nature and role of logic that is evident in this particular dialectic may be pervasive in legal theory—leading some theorists to reject logic as a source of useful insight in legal theory, and leading other to reject familiar and pervasive forms of reasoning because they do not conform to classical logic.[12] Logic should not be viewed as a doctrine, but as an *organon*—a tool, or set of tools: a method. When we are faced with a stable, coherent practice—such as the law—the proper role of logic is to develop and deploy the tools necessary for providing an illuminating formal understanding of that practice, not to force that practice to fit the straightjacket of classical logic.

Finally, my suggestion. In exploring *a contrario* reasoning, Duarte d'Almeida draws extensively on literature from the field of informal argumentation. This is a field that is sometimes held in disdain by analytic philosophers, and especially by logicians. I do not share that disdain—I think that the intricate analysis of actual human arguments found in

[11] Jansen (2003, p. 557).

[12] Theorists who reject logic as a useful source of insight in the law are too numerous to mention; theorists who reject pervasive forms of reasoning that do not conform to classical logic include, as an important example, Alexander and Sherwin (2008), who argue, in their work on precedential constraint, that the familiar practice of "distinguishing" is illegitimate, since it clashes with the picture of case rules as ordinary rules governed by classical logic.

this literature has a lot to teach philosophy, logic, and most likely legal theory as well. What I would like to suggest, however, is that theorists who are interested in legal argument should look, not just to the field of informal argumentation, but also to the field of AI and Law, as well as the closely-related field of formal argumentation. There they will find computational models of legal reasoning that are rich, detailed, mathematically precise, often implemented, sometimes experimentally validated.[13] Ideally, we can hope for a future in which legal theory is shaped by research from AI and Law in the same way that it has been shaped by work from fields such as cognitive science, economics, and public policy.

References

Aleven, Vincent and Ashley, Kevin (1997). Evaluating a learning environment for case-based argumentation skills. In *Proceedings of the Sixth International Conference on Artificial Intelligence and Law (ICAIL-97)*, pages 170–179. The Association for Computing Machinery Press.

Alexander, Larry and Sherwin, Emily (2008). *Demystifying Legal Reasoning.* Cambridge University Press.

Ashley, Kevin (1990). *Modeling Legal Argument: Reasoning with Cases and Hypotheticals.* The MIT Press.

Brewka, Gerhard (1994). Reasoning about priorities in default logic. In *Proceedings of the Twelveth National Conference on Artificial Intelligence (AAAI-94)*, pages 940–945. AAAI/MIT Press.

[13]As an example of experimental validation: the ground-breaking analysis of legal argument first suggested in Rissland and Ashley (1987), then presented most carefully in Ashley (1990), later served as the basis of an intelligent tutoring system for teaching elementary skills in legal argumentation, which achieved results comparable to traditional methods of instruction in controlled studies; see Aleven and Ashley (1997) for details.

Clark, Keith (1978). Negation as failure. In Gallaire, Hervre and Minker, Jack, editors, *Logic and Data Bases*, pages 293–322. Plenum Publishing Corporation.

Duarte d'Almeida, Luís (2021). Arguing *a contrario*. Unpublished manuscript.

Dung, Phan Minh (2016). An axiomatic analysis of structured argumentation with priorities. *Artificial Intelligence*, 231:107–150.

Gordon, Thomas (1993). *The Pleadings Game: An Artificial-Intelligence Model of Procedural Justice*. PhD thesis, Technische Hochschule Darmstadt.

Hansen, Jörg (2008). Prioritized conditional imperatives: problems and a new proposal. *Autonomous Agents and Multi-agent Systems*, pages 11–35.

Horty, John (2007). Defaults with priorities. *Journal of Philosophical Logic*, 36:367–413.

Horty, John (2012). *Reasons as Defaults*. Oxford University Press.

Jansen, Henrike (2003). *E contrario* reasoning and its legal consequences. In *Proceedings of the Fifth Conference of the International Society for the Study of Argumentation*, pages 557–559. Sicsat, Amsterdam.

Liao, Beishui, Oren, Nir, van der Torre, Leendert, and Villata, Serena (2019). Prioritized norms in formal argumentation. *Journal of Logic and Computation*, 29:215–240.

Prakken, Henry and Sartor, Giovanni (1996). A dialectical model of assessing conflicting arguments in legal reasoning. *Artificial Intelligence and Law*, 4:331–368.

Prakken, Henry and Sartor, Giovanni (1998). Modelling reasoning with precedents in a formal dialogue game. *Artificial Intelligence and Law*, 6:231–287.

Reiter, Raymond (1978). On closed world data bases. In Gallaire, Hervre and Minker, Jack, editors, *Logic and Data Bases*, pages 119–140. Plenum Publishing Corporation.

Rissland, Edwina and Ashley, Kevin (1987). A case-based system for trade secrets law. In *Proceedings of the First International Conference on Artificial Intelligence and Law (ICAIL-87)*, pages 60–66. The Association for Computing Machinery Press.

Scalia, Antonin (1997). *A Matter of Interpretation: Federal Courts and the Law*. Princeton University Press.