Homework I

1. (Camera models) For a camera, the image x of a point X in space is given by: $x = PX$, with x and X homogeneous 3 and 4-vectors respectively.

(a) If $PC = 0$, where C a homogeneous 4-vector, show that C is the camera center.

(b) The camera projection matrix P can be written as : $P = KR[I \mid -\tilde{C}]$, where K is the calibration matrix, R the rotation between the camera and world coordinate frames, and the camera center C is expressed as $C = (\tilde{C}, 1)$. Show that the calibration matrix can be obtained from a RQ decomposition of the first 3X3 sub matrix of the camera matrix P.

(c) In some application we obtained the following camera matrix:

$$P = \begin{matrix} 707 & 679.2 & 555.4 & -2898920 \\ 207 & 46.6 & 919.2 & -6325250 \\ 1.4 & -0.7 & 1.22 & -1837 \end{matrix}.$$

Find the camera center and the calibration parameters.

2. (Affine and metric rectification) Consider the image posted in the class web page. Read the image into matlab and use matlab to perform the calculations necessary for the following questions:

(a) Calculate two vanishing points.

(b) Find the image of the line at infinity

(c) Calculate a homography that maps the line at infinity to its canonical position. This homography produces an affine rectification. Apply this homography to the image.

(d) Calculate the dual conic.

(e) Use it to produce a metric rectification of the image.

3. (Planar mosaics) Consider the images posted in the class web site. They are images of a scene taken by a camera only rotating around an axis passing through its center. Thus any two of the images are separated by a rotation. Consider a point X in space and its images x and x' in two of the images. Then, assuming that the first camera coordinate system coincides with the world coordinate system, we have: $x = K[I \mid 0]X, x' = K[R \mid 0]X = KRK^{-1}x$. Thus, $x' = Hx$, with $H = KRK^{-1}$. H is called a conjugate rotation.

(a) Show that H and R have the same eigenvalues up to scale (one real and two complex).

(b) Show that the eigenvector corresponding to the real eigenvalue is the vanishing point of the rotation axis.
(c) Show that H is the infinite homography (i.e. the homography induced by the plane at infinity).

(d) Use the images provided to produce a planar panoramic mosaic.

4. (Projective reconstruction using coplanar points). Assume two sets of four known coplanar points in the scene {A,B,C,D} and {E,F,G,H} giving images {a,b,c,d} and {e,f,g,h} for an non calibrated camera with camera center O. Consider an unknown point M in the scene producing an image m.
(a) Show how to determine the viewing ray Om.
(b) Use (a) to develop a method for finding the camera center.
(c) Repeat (a) and (b) for the case where we are given 6 points in space and their images (instead of two sets of four coplanar points)