HOMEWORK 3

1. Suppose that you want to determine a hand eye transform, that is a transformation from the coordinate system of a mechanical manipulator to that of an electronic camera. This is a problem of exterior orientation. Assume that the camera system can determine the position of the image of a special mark on the gripper of the robot arm. You are free to command the arm to move to any position in its workspace. The transformation, as usual can be broken down into a translation and a rotation. Show that you need at least four calibration points to determine the transformation.

2. In edge based stereo methods, edges in the left image are matched with edges in the right image to obtain disparity measurements. Here we limit ourselves to matching along a single epipolar line, thus reducing the problem to one of matching in one dimension

a Suppose there are n edges in each image along an epipolar line. If each edge has a unique match in the other image, how many different mappings are there? Do not include the constraint that edges must be ordered the same way in both images

b Now add the constraint that edges must be ordered the same way in both images. If every edge has a unique match, how many different mappings are there?

c Now let the right image have m edges m < n. Then n - m of the left edges will be matched with the null edge. How many different mappings are there if we do not require that order be preserved?

d Repeat part c for the case in which the edges are ordered the same way in both images.

e Now consider the case in which there are n edges in the left image and m edges in the right image, and any edge can either have a match in the other image or not. How many different mappings are there with and without the ordering constraint?

3. In reasoning about points, lines and planes in space, it is often useful to use a construct called dual space. The point a in object space corresponds to the plane with equation $a \bullet v = 1$, in dual space. Symmetrically, the plane with equation $a \bullet v = 1$ in object space maps into the point a in dual space. Note that the dual of the dual of a point or a plane is the point or plane itself.

a Show that the unit normal of the plane $a \bullet v = 1$ is $\tilde{a} = \frac{a}{|a|}$ and that the perpendicular distance of the plane from the origin is $1/sqrt(a \bullet a)$

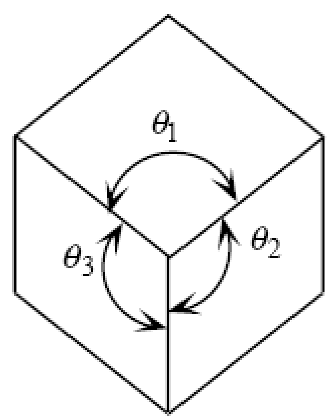
b A line can be defined either by two points it passes through or as the intersection of two planes that contain it. What is the dual of the line connecting the two points a_1, a_2? What is the dual of the line formed by the intersection of the two planes $a_1 \bullet v = 1, a_2 \bullet v = 1$

c Now superimpose dual space on object space. Show that a line is perpendicular to its dual.

4. In the case of pure rotation, the optical flow consists of vectors whose lengths do not depend on the distances of the objects. Instead, the magnitudes and directions of the vectors are determined by the axis of rotation and the angular velocity. Define the center of rotation to be the point where the optical flow is zero. Show that in the case of pure rotation, the motion can be determined from the optical flow at two points. Demonstrate further that at one of these points we need to know only the direction of the flow. Explain why this may not be the best way to use optical flow information.

5. a. Show that the perspective image of an ellipse lying on a plane, not necessarily parallel to the image plane, is also an ellipse.
b. Show that the image of a sphere is an ellipse whose major axis passes through the origin of the image. Can you use this result for calibration? How?

6. Consider the orthographic image of a trihedral vertex of a rectangular polyhe dron as in the figure below, where the angles $\theta_1, \theta_2, \theta_3$ are known. Determine the orientation of the polyhedron in space.



7. Investigate the following uniqueness issue. Consider an image motion field due to rigid motion of the observer.
(a) Is it possible that there are different rigid motions and surfaces which produce the same flow field? If yes, what are the constraints on the surfaces in view?

Hint: Suppose we have a motion $\{\mathbf{t}_1, \boldsymbol{\omega}_1\}$ and depth surface $Z_1(x, y)$ that yields the same motion field as a motion $\{\mathbf{t}_2, \boldsymbol{\omega}_2\}$ and a depth surface $Z_2(x, y)$.
We equate the two motion fields and obtain

$$\frac{1}{Z_1}(\hat{\mathbf{z}} \times (\mathbf{t}_1 \times \mathbf{r})) - \frac{1}{Z_2}(\hat{\mathbf{z}} \times (\mathbf{t}_2 \times \mathbf{r})) = \hat{\mathbf{z}} \times (\mathbf{r} \times (\delta\boldsymbol{\omega} \times \mathbf{r}))$$

where $\delta\boldsymbol{\omega} = \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1$.
Take the product of this vector equation with $(\mathbf{t}_1 \times \mathbf{r})$ and with $(\mathbf{t}_2 \times \mathbf{r})$ to obtain two scalar equations. From these equations the surfaces can be obtained. What are these ambiguous surfaces?

(b) If we consider an infinitely large image plane do these surfaces have positive value everywhere (that is, are they in front of the camera)?

8.
Consider an observer moving with rigid motion $\mathbf{t} = (U, V, W)$ and $\boldsymbol{\omega} = (\alpha, \beta, \gamma)$.

(a) The surface being imaged is a plane described as $Z = Z_0 + pX + q$ where X, Y, Z are the 3D coordinates of points on the surface. Show that

$$\frac{Z_0}{Z} = 1 - px - qy$$

where x and y are image coordinates.

(b) Develop the equations for the motion field (u, v) in terms of the motion and scene parameters.

(c) For this case, the motion parameters are not determined uniquely. Consider two different sets of rigid motions and corresponding planar surfaces, $(\mathbf{t}_1, \boldsymbol{\omega}_1, Z_{0_1}, p_1, q_1)$ and $(\mathbf{t}_2, \boldsymbol{\omega}_2, Z_{0_2}, p_2, q_2)$. How are these sets of parameters related to each other?

9. Compute the normal flow field for theYosemite Sequence using the method of Lucas and Kanade as implemented in Barron et al. (1994), which consists of the following steps:
(a) Filtering of the image sequence with a spatiotemporal Gaussian filter, with standard deviation sigma = 1.5 and
kernel size 11X11X11.
(b) Estimation of the spatial and temporal derivatives with a 5-point symmetric kernel $1/12 \times (-1, 8, 0, 8, -1)$ applied to the blurred image.
(c) Evaluation of normal flow values at points with high spatial gradient.
Thus to compute one flow field 15 images are required.

10.

2	3	4	5	6
3	4	5	6	8
4	5	$\boxed{6}$	8	5
5	7	8	9	3
9	10	9	4	3

Figure 1: Image patch

(a) For the image patch in Figure 1 at the pixel at the center (that is the pixel marked by the black square) apply the following filters and round to the nearest integer value:

i. a 3×3 Gaussian filter $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

ii. a 3×3 Box filter (that is averaging in a 3×3 neighborhood).

(b) Compute the edge direction and strength (that is the direction and absolute value of the image gradient) at the center pixel using the masks of the Sobel edge detector.

$$S_1 = \frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad S_2 = \frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(c) Apply a median filter to the center pixel. Explain for what kind of noise median filtering will work best.

(d) Why is the Gaussian filter a good smoothing filter? (How can it be implemented fast? How can we implement repeated Gaussian filtering in one operation?)

(e) What happens to the two edges at the boundaries of a dark line on a white background if the image is smoothed with a Gaussian with kernel size larger than the width of the line?

(f) Explain why Box filtering (that is averaging) attenuates the noise.