Problem

Let $f: \mathbb{Z} \to \mathbb{Z}$ be a function satisfying the following two conditions:

1. $f(n + a + b) - f(n + a) - f(n + b) + f(n) \in \{-1, 0, 1\}$ for all $n$, $a$, and $b$.

2. For all $n$ and $a$, there are an infinite number of values for $b$ such that $f(n + a + b) - f(n + a) - f(n + b) + f(n) = 0$.

Show that for some pair of reals $(x, y)$, either $f(n) = \lfloor xn + y \rfloor$ for all $n$, or $f(n) = \lceil xn + y \rceil$ for all $n$.

(Math Problem of the Week, 8/18/96)
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