Problem

Let $L$ be the set of lattice points on a 2-dimensional Cartesian coordinate system. We shall call a planar line a “lattice line” if it passes through an infinite number of lattice points.

(a) Determine whether there exists a subset $S \subseteq L$ such that every lattice line passes through exactly 2 members of $S$.

(b) Show that there exists a function $f: L \rightarrow [0,1]$ such that for any lattice line $Q$, the sum of $f(a,b)$ over all points $(a,b)$ on $Q$ is 1.

(c) Determine whether there exists a function satisfying the above conditions with $f(a,b) > 0$ for all $(a,b)$.

(Math Problem of the Week, 7/21/96)
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