Problem Set #3

Due date: Wednesday, March 4th.

1. Let $C \subseteq (\mathbb{F}_2)^5$ be the linear code with generator matrix

$$M = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$ 

Find the weight enumerator for $C$.

2. Let $K = \mathbb{F}_3[X]/(X^3 + 2X + 1)$, and let $\alpha$ be the element represented by $X$ in $K$. Compute the following. Express your answers as linear combinations of 1, $\alpha$, and $\alpha^2$.

   (a) $\alpha^4$
   (b) $\alpha^6$
   (c) $\alpha^{-1}$

3. Let $C \subseteq (\mathbb{F}_5)^4$ be the one-dimensional subspace generated by the vector $(1, 2, 2, 3)$. Find a generator matrix in standard form for $C^\perp$.

4. Let $G$ be the group of bijective maps

$$f: \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$$

such that $f(1) = 1$. (Multiplication in this group is given by composition.)

   (a) Is this an abelian group?
   (b) What is $|G|$?
   (c) What is the identity in this group?
   (d) Let $e \in G$ denote the identity. Find an example of an element $f \in G$ such that $f \neq e$ but $f^2 = e$.

5. Let $F$ be a finite field, and let

$$S = \{a^3 \mid a \in F\}$$

($S$ is the set of cubes in $F$.) How large is $S$ if $F = \mathbb{F}_{131}$? If $F = \mathbb{F}_{109}$? If $F = \mathbb{F}_{125}$? Explain your reasoning.

6. (Extra credit) Find the weight enumerator for a binary Hamming code of length $2^k - 1$. 