Problem Set #2

Due date: Wednesday, February 11th.

1. Find a self-dual binary [6, 3]-code.

2. Suppose that $n$ and $d$ are positive integers with $d > 2n/3$. Prove that does not exist a binary $[n, 2, d]$-code.

3. Suppose that a message consisting of 6 zeroes is sent through a binary symmetric channel with bit-error probability $p = 0.1$. What’s the probability the received message contains exactly three 0’s and three 1’s?

4. (a) Let $C \subseteq \mathbb{F}_2^7$ be the code with generator matrix

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix}
\]

Find a parity check matrix for $C$.

(b) Suppose that this code is used on a binary symmetric channel. Suppose that a 7-bit codeword $x \in C$ is sent across the channel, and the received word $x'$ is the same as $x$ except that the 5th bit is flipped. What’s the syndrome of $x'$? (Use the parity check matrix you found in part (a).)

5. Prove that there is no such thing as a perfect binary $[16, k, 3]$-code.

6. (Extra credit.) How many binary $[n, 2]$-codes exist? (In other words, count the number of subspaces of dimension 2 in $\mathbb{F}_2^n$. Express your answer as a formula involving $n$.)