Final Exam - Math 425

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Name: ________________________________

Please circle your answers. Cross out any work that you do not want graded.

If your answer is a decimal, write at least 3 significant figures.

Problem	**Grade**	**Max**
1		10
2		10
3		10
4		15
5		15
6		15
7		15
8		15
Total		105

Credit for problems 1-5 is based on your answers only.

(1) Mark each of the following statements as either "TRUE" or "FALSE."

- For any positive integers n and k,

$$\binom{n}{k} = \frac{(n+k)!}{k!}.$$

- If A and B are independent events, then $P(A \cap B)$ must equal $P(A)P(B)$.

- Let X be a continuous random variable with density function $f_X(x)$. Then for any real number a,

$$P\{X \leq a\} = \int_{-\infty}^{a} f_X(x)dx.$$

- Suppose that Y is a random variable which is uniformly distributed on an interval (a, b). Then the expected value of Y must be equal to $(b - a)$.

- For any random variable Z,

$$Var(-Z) = -Var(Z).$$

(2) Mark each of the following statements as either "TRUE" or "FALSE."

- Suppose that $p(x, y)$ is a joint probability mass function for two independent random variables X and Y. Then

  $$p(a, b) = p_X(a)p_Y(b)$$

  for any real numbers a and b. (Here $p_X(x)$ and $p_Y(y)$ are the mass functions for X and Y.)

- Suppose that $F(z)$ is the cumulative distribution function for a continuous random variable Z. Then the equation

  $$\lim_{z\to\infty} F(z) = 1$$

  must hold.

- If W is a continuous random variable, then $P\{W = w\} = 0$ for any real number w.

- The sum of two independent Poisson random variables is always a Poisson random variable.

- For any two random variables X_1 and X_2 on the same sample space, $E[X_1X_2] = E[X_1]E[X_2]$.

(3) Circle your answers to the following problems:

- A six-sided die is rolled repeatedly until a "1" appears. Let X be the total number of rolls in this experiment. Compute $P\{X = 4\}$.

- A stack of 10 cards (labeled "1," "2," ... "10") is shuffled at random. What's the probability that the "1" card and the "10" card end up next to one another in the stack?

(4) Circle your answers to the following problems:

- Let W be a normal random variable with $E[W] = 1$, $Var(W) = 4$. What is the density function for W?

- Let X be a continuous random variable whose density function is given by
$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$
Let $Y = 2X$. What is the density function for Y?

- Let Z be a discrete random variable whose probability mass function is given by
$$p(z) = \begin{cases} 1/3 & \text{if } z = 0, 1, \text{ or } 2, \\ 0 & \text{otherwise.} \end{cases}$$
Compute the variance of Z.

(5) Let X be a standard normal random variable. (In other words, X is a normal random variable which satisfies $E[X] = 0$ and $Var(X) = 1$.) Let Y be another standard normal random variable which is independent of X. Estimate the following quantities:

(a) $P\{0.5 < X < 1\}$

(b) $P\{X > 2 \mid X > 1\}$

(c) $P\{|X - Y| < 1\}$

Credit for problems 6-8 is based on your answers and on work shown.

(6) Suppose that a company employs two typists, Alice and Bob. Alice makes 0.3 typos per page (on average) and Bob makes 0.5 typos per page (on average).

Suppose that I give the company a handwritten draft of a book and ask them to type it for me. When typed, the book will consist of 100 pages. The company chooses either Alice or Bob to do the typing based on a fair coin flip.

(a) What's the expected number of typos in my book?

(b) Suppose that I get my book back and I find that it has 35 typos. Based on this information, what's the probability that Alice did the typing? (State any assumptions that you make.)

(7) Charlie and David are scheduled to meet at a coffee shop at around noon. Suppose that Charlie's arrival time is uniformly distributed between 12:00pm and 12:10pm, and David's arrival time is uniformly distributed (independently) between 11:50am and 12:10pm.

(a) What's the probability that both men will arrive by 12:05pm?

(b) Let X be time (measured in minutes past 12:00pm) when the two men actually meet one another at the coffee shop. What's the density function for X?

(8) 15 men are seated around a large round table in random order. Each man compares his age with the man immediately to his left and the man immediately to his right. A man is considered "senior" if he is older than both of his neighbors. Let X be the number of "senior" men at the table.

(We'll assume that no two men are exactly the same age.)

(a) Compute $E[X]$.

(b) *(Extra credit)* Compute $Var(X)$.

(Attach normal random variable table.)