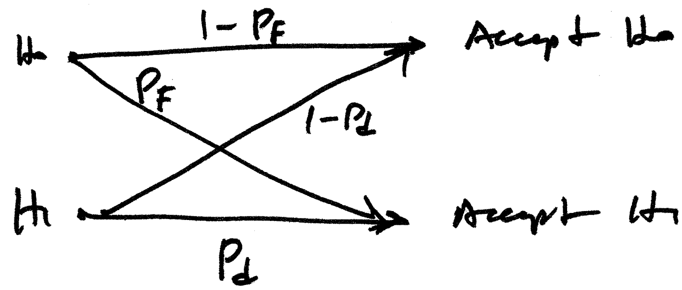
HW #1

Due 18 September.

(20 PTS) ① A binary hypothesis testing problem can be described as a communication channel where the inputs are the hypotheses and the outputs are the decisions. The transition probabilities are related to the false-alarm (P_F) and detection (P_d) probabilities.



Define

$$I(\underline{x}; \underline{y}) = H(\underline{x}) - H(\underline{x} \mid \underline{y}), \qquad I: \text{MUTUAL INFORMATION}$$

$$H(\underline{x}) = -\sum_i P(x_i) \log P(x_i),$$

$$H(\underline{x} \mid \underline{y}) = -\sum_{i,J} P(x_i, y_J) \log \frac{P(x_i, y_J)}{P(y_J)},$$

where $P(x_i)$ denotes the a priori probabilities, $P(y_i)$ the output probabilities, and $P(x_i, y_J)$ the joint probability of input x_i, resulting in output y_J.

For example, $P(x_0, y_0) = P(x_0) \cdot (1 - P_F)$

$$P(y_0) = P(x_0)(1 - P_F) + P(x_1)(1 - P_D)$$

Then, given $P(x_0)$ and $P(y_0)$, what is the decision rule that maximizes the mutual information?

(5 PTS EACH) ② (DHS Chapter 2) Problems 10, 26, 32, 47

(20 PTS) ③ (DHS Chapter 2) Computer Exercise 7