The Design of the Multi-Attribute English Auction with a Deadline Vs. the eBay Auction Protocol[1]

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ABSTRACT

This paper provides an optimal auction mechanism design for automated agents conducting auctions, for example, auction houses on the web. In particular, we consider auction protocols that include two major features; multi-attribute items and a deadline rule. Three different protocols were analyzed. For each protocol a mechanism was developed that enables the auctioneer agent to compose an optimal scoring function based on its expected revenue's estimation. Two of these protocols; the *simultaneous* and the *sequential* English protocols, were defined in previous work. The incentive of developing these protocols was to overcome the drawback of available auction protocols with a deadline that encourage the anomalous bidders behavior of applying the "last minute bidding strategy" as in eBay and Amazon auction houses. The third protocol we analyzed is the eBay protocol, which is an example of a widespread auction house. We proved that the eBay protocol is always dominated by the simultaneous protocol. Furthermore we showed that there are cases where the sequential protocol achieves better results with respect to the eBay and the simultaneous protocols. Consequently auction houses are motivated to employ one of the two protocols we defined in cases where the probability of losing bids at the last minute is not negligible.

Categories and Subject Descriptors

Intelligent agents, Multiagent systems.

General Terms

Economics.

Keywords

Electronic commerce market, Deadline rule, Auctioneer agent.

1. INTRODUCTION

In many real world situations it is necessary to terminate a negotiation among agents and to reach an agreement within a predefined deadline. For example, consider the supply request in the supply chain domain in which, it is understandable that the supply should be provided within a fixed deadline. Since there may be many dependencies among the involved markets, a delay in one market will cause a consequent delay in production and negotiations in other dependent markets. In conclusion, a deadline feature should be enabled in each auction or negotiation protocol.

In addition we consider a multi-attribute auction, which is a protocol that enables bidding over multiple dimensions of an item [2,3]. Bargaining agents can effectively employ a multi-attribute auction mechanism to reach an agreement consisting of several negotiable dimensions [9,10]. Usually, one dimension is the price and the others are quality dimensions. To illustrate a multi-attribute item consider the following examples. In the resource allocation problem a resource may be described by its price, speed, computation power, accuracy, etc. Another example can be a request for supply in the supply chain domain. Usually such a request is characterized by several attributes such as the type of goods, the amount, the supply time, the transportation means and other informative and qualitative attributes. A few researchers have considered multi-attribute auctions but they did not consider the deadline feature [1,2,3,9,10]. One may think that the multi-dimensions can be easily mapped to a single dimension using a mapping function. However, the main problematic issue is to find the optimal mapping function (scoring-function), which is not necessarily the real utility function of the auctioneer. Moreover, given the scoring function, it is still non trivial for the bidder to identify the optimal bid (for more details see [4,5]).

In previous work we defined two multi-attribute auction protocols with a deadline rule; the *simultaneous* and the *sequential multi-attribute English auctions with a deadline* [6]. We were motivated to define new deadline rules to overcome the anomalous behavior

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of the last minute bidding in the widespread auction protocols that enable deadline rules such as eBay and Amazon protocols. The last minute bidding causes an overhead for the system in both types of deadline rules used in eBay and Amazon. Moreover, in eBay it may also cause an inefficient outcome resulting in good bids missing the deadline and therefore not being considered. Even though it is believed that the last minute bidding occurs in common value models it has been proven by [8] that it occurs also in private value models, which we refer to. Ockenfels and Roth [7] conduct a in-depth discussion and an experimental analysis on the reasons that cause people to use the last minute bidding strategy.

The principles of our deadline rules are: (1) defining the number of rounds for the auction and the order of bidding in each round of the auction, (2) defining the last round in such a way that no new bidder is allowed to join the auction, (3) defining the duration of a round to be long enough to ensure the acceptance of the permitted bids.

In previous work [6], we provided the bidders in these auctions (simultaneous and sequential) with an optimal bidding strategy that results in calculated behavior, which can be applied by autonomous agents.

In this paper we explicitly express the auctioneer's expected revenue and present the optimal scoring function that yields the best expected-revenue for the sequential and simultaneous protocols with a deadline. The next interesting question we aim to answer is whether our protocols with the deadline rule achieve better outcomes with respect to the un-recommended behavior of last minute bidding which occurs, for example in the eBay auction. For this reason we provide a full analysis of the eBay protocol for the case of multi-attribute items in a reverse auction where bidders assumed to follow the common equilibrium bidding strategy of the last minute bidding strategy. We compare the three protocols (simultaneous, sequential and eBay) with regard to the auctioneer's expected revenue. Our findings show that the simultaneous protocol always achieves better or equal results with regard to the eBay protocol, assuming one common bidding strategy which is in equilibrium is applied. Moreover, in several cases the sequential protocol also achieves better expected-revenue than the eBay protocol. In conclusion we found that in each case, not only did our protocols overcome the un-recommended bidding behavior, but at least one of the protocols we defined also achieved better results for the auctioneer. Notice that the results of this paper hold also for the private case of single attribute items.

2. THE MODEL

The model described in this section was considered in previous works [4,5,6]. The auction model consists of one auctioneer agent and several bidder agents. The auctioneer agent that needs a particular item (service or product) starts the auction process. At the beginning of the auction, the auctioneer announces its item request, which consists of the item's desired characteristics, the auction protocol, and a scoring rule describing its preferences concerning the item properties. A bidder agent that decides to send a bid has to specify the full configuration it offers.

The scoring function associates a score with each proposed bid and the auction protocol dictates the winner and the winning bid. The scoring function is used by the auctioneer as a tool for choosing among a set of offers. On the other hand, the bidders use the scoring function to optimally bid. Therefore, the auctioneer agent is motivated to derive a scoring function that maximizes its expected utility in a given auction protocol. We assume that each participant knows its utility function, and time and bidding are not costly.

Each auctioneer agent and each bidder agent is characterized by a utility function that describes its preferences. In practice it is not a trivial task to draw a utility function. There are numerical methods [13] and software that guide users to construct and design an additive utility function based on their past experiment. Therefore, they are common used functions and we follow this tradition. The multi-attribute utility-functions we refer to are based on the Simple Additive Weighting (SAW) method [11]. A utility or a score in the SAW method is obtained by adding the contributions of each attribute (linear function). There are other methods used for multi-attribute utility functions (e.g. multiplying the contributions of the various attributes). An SAW function may, for example, suit the case of a request in the supply chain domain.

In our model, each bidder agent has private information about the costs of improving the quality of the product it bids, or its performance. Each bidder agent B_i (bidder) is assumed to be characterized by a cost parameter θ_i, which is its private information. As θ_i increases, the cost of the bidder to achieve an item of a higher quality also increases, i.e., the bidder is "weaker". The auctioneer only knows the distribution function of the other bidders' cost parameters, and has no information about the particular value of θ_i of each bidder.

Similar to the model described by Che [3], we assume that θ_i is independently and identically distributed over $[\underline{\theta},\bar{\theta}]$ ($0<\underline{\theta}<\bar{\theta}<\infty$) according to a distribution function F for which a positive, continuously differentiable density f exists. Because of complete symmetry among agents, the subscript i is omitted in the rest of the paper.

We analyze a general case of a multi-attribute auction in which there is an arbitrary number of attributes (m+1), which is predefined and known to all the participants. One of the attributes is the price (p) and the others are quality attributes (q_i where $i \in [1,\dots,m]$) for which the preferences of the auctioneer and the bidders are opposite. We assume that as q_i increases, the quality of the item increases. That is, as q_i increases the cost of the bidder to provide it increases since it is harder to provide higher quality items. In addition, the auctioneer's utility from a higher quality item increases.

Consider the cost functions of the bidders. We assume that there are fixed coefficients for each of the quality dimensions which are identical for all the bidders. Namely, a_1 is the coefficient of q_1, and a_2 is the coefficient of q_2 and similarly a_i is the coefficient of quality attribute q_i.

The extension in a sense of having a vector of cost parameters per bidder that indicates the cost parameter for each quality attribute is possible as we showed in [5] for a particular protocol. However, this assumption complicates the calculation and enables us to reach only general results. Whereas, given specific values of the parameters actual results can be calculated. Therefore, for simplicity reason, in this paper we assume that each bidder is characterized by one cost parameter.

The bidder's cost function is: $C_B(q_1,...,q_m,\theta) = \theta\left(\sum_{i=0}^{m} a_i \cdot q_i\right)$, where $a_i > 0$. Based on the cost function, the bidder's utility function is: $U_B(p,q_1,...,q_m,\theta) = p - \theta \cdot \left(\sum_{i=0}^{m} a_i \cdot q_i\right)$.

We assume that the utility function of the auctioneer agent from an item is as follows: $U_{auc}(p,q_1,...,q_m) = -p + \sum_{i=1}^{m} W_i \cdot \sqrt{q_i}$, where W_i are the weights the auctioneer associates with q_i, respectively. Remember that, the price is what the auctioneer pays in exchange for the received item or service. Consequently, as the price decreases the auctioneer's utility increases and as q_i increases the utility of the auctioneer increases. We assume that the q_i's where $i \in [1,...,m]$ are independent but not linear: as q_i increases, the influence of one additional unit of q_i becomes less. This assumption is valid in many domains. For example, enlarging the speed of a machine from 100 Mhz to 200 Mhz will have a greater influence than enlarging the speed from 200 Mhz to 300 Mhz. The effect of q_i is weighted by W_i, respectively, where W_i can be smaller or larger than 1. As W_i increases the importance of attribute q_i to the auctioneer increases, w.r.t. the price and the other attributes.

Given the auctioneer's utility function, the auctioneer will announce a scoring function, which is used for choosing among bids. In particular, the scoring function is of the form: $S(p,q_1,...,q_m) = -p + \sum_{i=1}^{m} w_i \cdot \sqrt{q_i}$, where w_i are the weights that the auctioneer assigns to q_i. The announced values of the weights w_i can be equal to or different from the real values of the weights W_i. For example, if $w_i < W_i$, then for some reason the auctioneer will declare a lower utility derived from each unit of q_i than its actual utility from q_i. In Sections 3.4 and 4.2 we will show how optimal announced weights can be determined.

3. THE MULTI-ATTRIBUTE ENGLISH AUCTION WITH A DEADLINE

In section 3.1 we describe the protocols of multi-attribute English-auctions with a deadline. We consider the sequential and the simultaneous protocols, which were defined in previous work [6]. We start by describing the common part of these two protocols and then we proceed by describing the specific part of each protocol. In Section 3.2 and 3.3 we describe the optimal bidding strategy for the first rounds and for the last round, respectively, as descried in previous work [6]. Given the protocols and the bidding strategy provided in [6], in section 3.4 we present, for the first time, the optimal auction design for the auctioneer using the simultaneous and the sequential protocols.

3.1 The Protocols

At the beginning of a multi-attribute English auction with a deadline the auctioneer announces: (1) a scoring function, $S(p,q_1,...,q_m)$ (2) the minimal increment allowed, D and (3) the maximum number of rounds that will take place till the auction is closed, R.

According to the principle of an English auction with respect to multi-attribute items, each placed bid's score should be higher than the score of the previous proposed bid by the minimal increment D. If a bidder agent prefers not to bid then it will not proceed to the next round and it will be considered to have dropped out of the auction. New bidders can join the auction in each of the R-1 rounds. Notice, we assume that autonomous agents participate in the auction on behalf of human bidders and the auctioneer. Therefore, the requirement of participating in each round at the assigned turn is reasonable. However if human bidders participate in such a protocol, a relaxation can be done in the protocol in such a way that the bidders only have to subscribe during the R-1 rounds and then participate in the last round. That is, for the R-1 rounds a flexible protocol variation can be applied as long as the bidders know the beginning and ending time of each round and that they may bid once in a round. However for the last round we require that the following protocols be used.

A sequential protocol with a deadline (SED): Each seller is allotted a serial number through a lottery at the beginning of each round. In each round, each seller can place a bid during its turn. The participant bidder agents observe the proposed bids at each step.

A simultaneous protocol with a deadline (SID): In each round, all the bidders bid simultaneously; the winning bid of each round is chosen and can be observed by the participating bidder agents. In the next round, each bid should exceed the score of the winning bid of the previous session. No bid except the winning bid is announced.

3.2 The Bidder's Strategy in the R-1 Rounds for the Sequential and the Simultaneous Protocols

In the first R-1 rounds, the bidder agents only have to indicate their participation in the auction and to follow the protocol, which is to increase the score of the next proposed bid by the minimal increment allowed, D. This is because the first R-1 rounds are actually used to gather the interested participants and to prepare the bidders for the last significant round. Moreover, the first round may deter weak bidders from continuing to participate. Notice that R should be a small number. In each of the R-1 rounds the bidders can improve their bids in the next round based on the previous best bid observed.

As was proved in [6], in each of the R-1 rounds, the optimal bidding strategy of bidder agents in SED and SID protocols is the same as in the multi-attribute English auction with no deadline which we provided in [5]. According to this optimal strategy, the bidders choose the quality attributes' values depending only on the announced scoring function and their cost parameter with no influence of the other participants. Then the bidders choose the price in such a way that the entire bid yields a score that is higher than the score of the previous selected bid by D.

The bidders that reach the last round have to change their strategy in order to ensure their winning, since they will not have another chance to improve their bid.

3.3 The Bidding Strategy in the Last Round of

The Sequential protocol:

In the last round of a sequential protocol the optimal strategy of the bidder as we proved in [5] is to consider (1) the last proposed bid, (2) the number of bidders positioned after him and (3) their estimated cost parameters (the affect of the bids on the bidders ahead of him is concealed in the last proposed bid). All the bidders excluding the last one should speculate and try to calculate the bid that maximizes their expected utility. The last bidder that sees all the others' bids only has to improve the last proposed bid by D if it can afford to. Thus its utility by proposing this bid is non-negative. We use $OptS(\theta, k)$ to denote the optimal score of a bidder with a cost parameter θ, where there are k bidders that may bid after him.

The Simultaneous Protocol:

As was proved in previous work [6] the bidders in a simultaneous protocol with a deadline use the dominant strategy of the first-score sealed-bid auction as specified in [4]. Following this strategy, the qualities' values are chosen based only on the scoring function and the private cost parameter. However, the price is determined based on speculation about the other n-1 participating sellers.

There is no way to avoid bidders' speculations where a real deadline rule is defined. In Amazon and Yahoo the deadline rule actually does not play any role in the sense that each bidder has the opportunity to improve his bid any time he wants. But in auctions with a strict deadline rule as in eBay, this is not the case and therefore speculations also in the eBay protocol are required.

The combination of the sequential and the simultaneous protocols where a certain protocol is used for the first R-1 rounds and another protocol is used for the last round in case all the bidders are allowed to participate in the last round follows the same results as the auction type of the last round. However if a combination of protocols is considered where only some of the bidders are allowed to proceed to the last round then further research should be conducted to analyze this protocol.

3.4 The Optimal Auction Design for the Sequential and the Simultaneous Protocols

For the auction design phase we assume that there are n bidders in the last round. The restriction of the number of bidders is used only for the last round of the sequential and the simultaneous protocols and for the last minute in the eBay protocol. This is a reasonable assumption for the Internet domain since the participant registers before the last round or the last minute**.**

The Sequential Protocol:

The auctioneer's Expected Revenue in the Sequential protocol with a Deadline, (ER^{SED}), is based on the idea behind the optimal bidding strategy (Section 3.3). Recall that according to the SED protocol each bidder can bid only on its turn. In the last round, the bidder speculates about the following bidders and tries its best with regard to its cost parameter (θ) and the information it assumes about the other bidders after it (k out of n). By using this information (θ, k) the bidder can optimally bid w.r.t the announced scoring function.

The estimation of the ER^{SED} made by the auctioneer, is the utility from the best-expected bid multiplied by its winning probability. Since the auctioneer has no specific information about the bidders' cost parameters, but only their range, the auctioneer calculates the average of all the possibilities of the winner's cost parameters and its possible turn which yields the best-expected bid for each set of parameters (θ, k).

The probability that a given bid wins, is the probability that its score is higher than the score of all the possible bids regarding the combination of θ and k, where $\theta \in [\underline{\theta}, \overline{\theta}]$ and $k \in [1,...,n]$.

Theorem 1

In a sequential English auction protocol with a deadline comprising one auctioneer agent and n bidder agents in the last round with cost parameters independently, identically and uniformly distributed over $[\underline{\theta}, \overline{\theta}]$*, given the form of the bidders' utility function (* a_i *where* $i \in [1,...,m]$ *), given the minimal increment D, given the real weights* W_t where, $t \in [1,...,m]$*, of the auctioneer's utility function, and the announced weights* w_t where $t \in [1,...,m]$ *of the scoring rule, the auctioneer's expected revenue* ER^{SED} *is:*

$$ER^{SED}(\underline{\theta}, \overline{\theta}, n) = \frac{1}{n-1} \cdot \sum_{k=1}^{n-1} \int_{\underline{\theta}}^{\overline{\theta}} Uauc(q*_1(\theta),...,q*_m(\theta), p*(\theta))$$

$$\cdot f(\theta) \cdot \prod_{i=1, i \neq k}^{n} Prob(OptS(\theta_i, i) < OptS(\theta, k)) d\theta.$$

where

- $\{q*_1(\theta),...,q*_m(\theta), p*(\theta)\}$ - the optimal bid of a bidder agent with a cost parameter θ.
- $f(\theta) = \frac{1}{\overline{\theta} - \underline{\theta}}$ -density function.
- $OptS(\theta, k) = -D - \frac{g(k-1)}{8 \cdot \overline{\theta}} + \frac{g}{8 \cdot \overline{\theta}} \cdot \sqrt{\frac{16 \cdot \overline{\theta} \cdot k \cdot D}{g} + (k-1)^2 + \frac{4 \cdot \overline{\theta} \cdot k}{\theta}}$

Where $g = \sum_{i=1}^{m} \frac{{w_i}^2}{a_i}$.

$$Prob(OptS(\theta_i, i) < OptS(\theta, k)) =$$

$$\frac{1}{\overline{\theta} - \underline{\theta}} \cdot \left(\frac{(4\overline{\theta} \cdot i)}{\left(i - k + \sqrt{\frac{16 \cdot \overline{\theta} \cdot k \cdot D}{g} + (k-1)^2 + \frac{4 \cdot \overline{\theta} \cdot k}{\theta}}\right)^2 - \frac{16 \cdot \overline{\theta} \cdot i \cdot D}{g} - (i-1)^2} - \underline{\theta} \right)$$

Sketch of Proof

The auctioneer tries to estimate his expected revenue by first considering the cost parameter of the winning bidder θ. Given the winner's cost parameter, the auctioneer calculates his utility from the winning bid, $U_{auc}(q*1(\theta),...,q*m(\theta), p*(\theta))$ assuming the bidders follow the bidding strategy described in [6]. This utility is multiplied by the probability of having a bidder with a cost parameter θ and all this multiplied by the probability that the bidder with a cost parameter θ wins. The probability a bidder winning, is the probability that this bidder proposes a bid with a higher score than all the other bidders considering their cost parameter and the number of bidders that may bid after it.

This probability is indicated by $\prod_{i=1,i\neq k}^{n} Prob(OptS(\theta i, i) < OptS(\theta, k)$. We calculate the average of all the auctioneer revenues considering the cost parameter range of the bidders $[\underline{\theta}, \bar{\theta}]$ by the integral ($\int_{\underline{\theta}}^{\bar{\theta}}$). For each given cost parameter of the winning bidder θ, we average all the possible turns (k) of the winning bidder ($\frac{1}{n-1} . \sum_{k=1}^{n-1}$).

■

Given the ER^{SED} function the auctioneer agent can identify the optimal weights to announce in the scoring function. In general, it is very complex to explicitly express the optimal weights. However, by knowing the parameters' values (D, $\underline{\theta}, \bar{\theta}$,n,m) and assigning them to the ER^{SED} the optimal weights can be found by solving a maximization problem of the ER^{SED} with regard to w_t, where $t \in [1,...,m]$ (using for example Maple, or optimization methods), or by observing the graph that presents the ER^{SED} values as a function of w_t , $t \in [1,...,m]$.
To illustrate the behavior of the ER^{SED} as a function of the announced weights w_t see Figure 1. For this illustration we set the parameters' values as: $m = 2, n = 3, D = 0.01,$ $\underline{\theta} = 0.1, \bar{\theta} = 1, a_1 = a_2 = 1, W_1 = W_1 = 10$.

As we see, there is a maximum value of the ER^{SED} located in (w_1 =6.8 w_2 =6.8). That is, for some values of the announced weights in the scoring rule, the ER^{SED} is maximized. Notice that the number of rounds has no affect on the auctioneer's expected revenue.

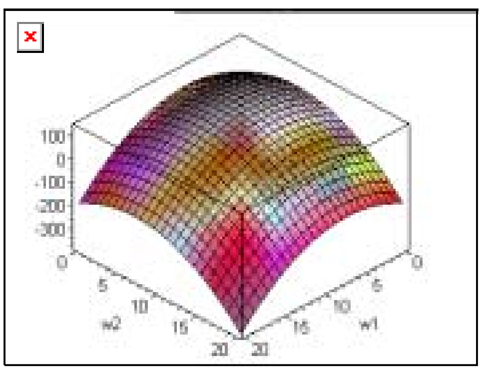

Figure 1. The ER^{SED} as a function of w_1 and w_2 .

Simultaneous Protocol:

Recall that according to the Simultaneous protocol with a Deadline (SID) in each round all the bidders have to submit their bid during the round and no one observes the other's proposed bids. As described in Section 3.3 the bidders at the last round act exactly as if they were participating in a first-score sealed bid auction. Therefore, the analysis of the auctioneer's expected revenue and the mechanism of finding the optimal scoring function are identical to those of the first-score sealed-bid protocol [4]. Due to space limitation the proofs are omitted but can be found in [12].

Lemma 1

$ER^{SID} = ER^1$ and the optimal weights, W_t where $t \in [1,...,m]$, of the scoring rule in the simultaneous protocol and in the first-score sealed-bid auction are equal.

Recall that the motivation to develop and to analyze multi-attribute auction protocols with deadline rules, was to overcome the un-recommended behavior of the "last minute bidding" strategy that occurs in common auction houses in the internet that involve a deadline feature. Once we have the full analysis that includes the optimal bidding strategy and the possibility to estimate the outcome of each protocol, we would like to compare the auction results of our proposed protocol with an example of auction houses that apply a deadline rule. In particular we choose the eBay protocol. In the following section we analyze this protocol and we provide a mechanism to estimate the outcome of such a protocol (the expected revenue function).

4. THE EBAY AUCTION PROTOCOL

eBay is an example of an auction house that uses a fixed deadline rule. The advantage of this protocol is that the exact time the auction closes is known in advance and encourages people's participation. On the other hand its disadvantage is that it encourages the last minute bidding strategy that causes system overhead [7] and misplaced bids submitted at closing time. Sometimes the reason that the best bidder waits until the last minute to bid his true value is to avoid a "bid war" which unnecessarily increases his and other subsequent bids [7].

In this section we analyze the eBay protocol for the case of a reverse auction with multi-attribute items and we provide the optimal auction design given the common bidding strategy that is used by most of eBay's users.

In order to formally analyze eBay's protocol we use the definition of the last minute (t=1) defined by [8]:

Definition 1 [8]

A player can bid at any time $t \in [0,1) \cup \{1\}$. A player has time before the end of the auction to react to another player's bid at time t'<1. At t=1, everyone knows the bid history prior to t, and has time to make exactly one more bid, without knowing what other last minute bids are being placed simultaneously.

Definition 2 [8]

♦ *Bids submitted at time t where $t \in [0,1)$ are successfully transmitted with certainty.*

♦ *At time t=1, the probability that a bid is successfully transmitted is p<1.*

eBay uses an English protocol for single attribute items in which each bidder has to propose a higher price, each time until the defined deadline {t=1). The highest bidder wins the auctioned item. In equilibrium the winner proposes a bid that is equal to the second highest bid plus some minimal increment. In each phase the winning price is published

We expand the use of the eBay protocol to the case of multi-attribute items. Accordingly, all the bids' values are not prices but scores of a specific bid configuration. The winner should provide the auctioneer with a configuration that is worth at least the score of the second highest bid.

Before we start the auction analysis we first emphasize the differences between the sequential & simultaneous English protocols and the eBay's protocol.

- The time according to eBay protocol can be considered a continuous parameter. However, according to our sequential & simultaneous protocols it is somehow much closer to being a discrete parameter. This is because the bidding time is divided into predefined sequences in which, each bidder has only one opportunity to bid (see Section 3.1 for more details).
- In eBay there is a strict deadline. However, in the sequential & simultaneous protocols there is a last round of bidding.
- In eBay the probability that a bid which was submitted at the last minute bidding, will reach the auctioneer, is p<1. Whereas according to the sequential & simultaneous protocols the probability that the proposed bid at the last round will reach the system is 1.
- According to eBay there is no bidding order. However according to the sequential & simultaneous English protocols with a deadline rule that we defined, a bidding order exists and it is a substantial part of the protocol.
- According to eBay the winner pays the second highest bid plus a minimal increment, and according to the sequential & simultaneous protocols the winner pays his proposed bid.

We summarize this discussion in Table 1.

Table 1. The Differences between eBay and SID & SED protocols

	Ebay	SED& SID
Time	Continuous	Discrete
Deadline	Strict	Last round
Probability of bidding at the last minute	P<1	P=1
Bidding order	No bidding order	Defined
Winning bid	The second highest scored plus minimal increment	The winner's proposed bid

4.1 Optimal Bidding Strategy in the eBay Auction

There are multiple bidding strategies for the eBay auction protocol, which are in equilibrium, including the equilibria at which each bidder bids his true value at t=0 (a true value means a value that achieves zero utility for the bidder with regard to the bidders' utility function) [8]. However, there are also equilibria at which no bidder bids his true value until the last minute [8]. Ockenfels and Roth[8], proved a theorem that provides an example of such an equilibrium of bidding the true value at the last minute (t=1).

As a consequence of the experimental evidence [7,8] showing that the majority of bidders use the last minute bidding strategy, we use this strategy as an optimal strategy for the next phase of calculating the auctioneer's expected revenue. We choose this equilibrium in order to compare the results of our protocols to something real that exists, such as eBay.

Lemma 2 [8]
One of equilibriums strategy in the eBay protocol is to bid the true value at t=1.

In the rest of the paper where we refer to eBay auction results we assume the equilibrium specified in Lemma 2.

Lemma 3
The optimal bidding following Lemma 2 of a bidder with a cost parameter θ, in multi-attribute eBay auctions is to bid the following specified bid at time t=1.

$$\left\{ p(\theta) = \frac{1}{4\cdot\theta}\cdot\sum_{t=1}^{m}\left(\frac{w_t}{a_t}\right)^2; q_t(\theta) = \left(\frac{w_t}{2\cdot a_t\cdot\theta}\right)^2 \right\} where, t \in [1,..,m]$$

Sketch of proof

The qualities are determined independently of the auction protocol [4]. The price is calculated by looking for the price that achieves zero utility for the bidders considering the optimal qualities.
■

In the following section we analyze the auction from the auctioneer's point of view. We calculate the auctioneer's expected revenue and give a method for optimally designing the announced scoring function in multi-attribute eBay auctions.

4.2 The Optimal eBay Auction Design

The following lemma specifies the auctioneer's expected revenue in a multi-attribute eBay auction.

Theorem 2
The auctioneer's expected revenue in a multi-attribute eBay auction protocol, where there are n participating bidders in the last round, the distribution of the bidders cost parameters are uniformly and identically distributed over $[\underline{\theta},\overline{\theta}]$, and where the probability that a bid which is proposed at t=1, successfully reaches the system is p, is:

$$ER^{eBay}(n,\underline{\theta},\overline{\theta},p) = (1-p)^n \cdot min\_s + C_n^1 \cdot (1-p)^{n-1} \cdot p \cdot min\_s + \sum_{i=2}^{n} C_n^i \cdot (1-p)^{n-i} \cdot p^i \cdot ER^1(i,\underline{\theta},\overline{\theta})$$

where min_s is the minimal score that the auctioneer requires, and $ER^1(i,\underline{\theta},\overline{\theta})$ is the expected revenue calculated for the English auction as specified in [4].

Sketch of proof
Assuming the bidders use an optimal bidding strategy as specified in Lemma 3, the expected revenue is calculated. Since p is assumed to be less than 1, there is a probability that the auctioneer will not receive all the proposed bids. Based on this, the auctioneer calculates the probability of the various cases (the number of received bids) multiplied by the expected revenue of each case. For each given case of i received bids at the last minute the auctioneer calculates the expected revenue based on the expected revenue of the first-score sealed-bid [4] with i bidders, multiplied by the probability of this case to happen which

is the product of: (1) the probability that the i bids reach the auctioneer, (2) the probability that the $n-i$ bids missed the deadline, and (3) the combination's number of i bidders out of n.

■

From the above we can conclude that the expected revenue of the eBay protocol is less than or equal to the expected revenue of the first-score sealed-bid auction, $ER^{eBay} \le ER^{1}$. The practical significance is, that in most cases, using the simultaneous English auction with a deadline is better than applying the eBay' protocol with regard to the auctioneer's outcome. In Lemma 1 we claimed that $ER^{SID} = ER^{1}$. Consequently, $ER^{eBay} \le ER^{SID}$.

Lemma 4

Given the equilibrium of Lemma 3, if p<1, $ER^{eBay} < ER^{SID}$ *and if p=1,* $ER^{eBay} = ER^{SID}$.

In Figure 2 we illustrate the behavior of ER^{eBay} as a function of p. As p increases, eBay becomes more efficient. In addition, we demonstrate that for p=1 $ER^{eBay} = ER^{SID}$. *The parameter values are:* $m = 2, n = 5, D = 0.01,$, $\bar{\theta} = 10$ $\underline{\theta} = 1, a_1 = a_2 = 1$ $W_1 = W_2 = w1 = w2 = 10, reserved = 0$.

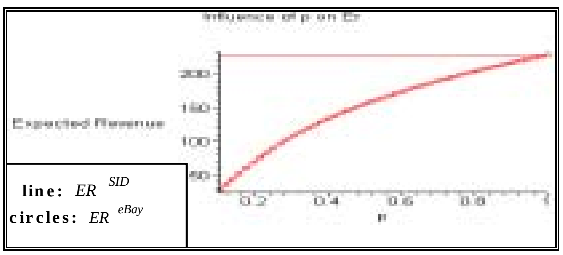

Figure 2. Expected revenue as a function of p.

Given the expected revenue function of the eBay's protocol, the auctioneer agent can find the optimal weights to be announced in the scoring function as indicated in Theorem 3.

Theorem 3

Given a multi-attribute eBay auction protocol of one auctioneer and n bidders in the last minute which are associated with independently and identically private cost parameters uniformly distributed over $[\underline{\theta}, \bar{\theta}]$ and given the real weights W_t where $t \in [1,...,m]$ of the buyer utility function, the optimal values of the announced weights w_t where $t \in [1,...,m]$ of the scoring rule are:

$$w_i\left(\underline{\theta}, \bar{\theta}, n\right) = W_i \bullet \frac{\sum_{i=2}^{n}\left[f(i) \bullet (1-i)\right]}{\sum_{i=2}^{n}\left[\left(2f(i) - g(i)\right) \bullet (1-i)\right]}$$ where:

$$m(i) = \frac{n! \cdot (1-p)^{n-i} \cdot p^{i}}{(i-1)! \cdot (n-i)! \cdot (\bar{\theta} - \underline{\theta})^{i}},$$

$$f(i) = m(i) \cdot \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{t} \cdot \int_{t}^{\bar{\theta}} (\bar{\theta} - z)^{i-2} dz dt,$$

$$g(i) = m(i) \cdot \int_{\underline{\theta}}^{\bar{\theta}} \int_{t}^{\bar{\theta}} \frac{(\bar{\theta} - z)^{i-2}}{z} dz dt.$$

Sketch of Proof

The optimal weights w_t can be found by solving a maximization problem of ER^{eBay} with regards to the announced weights w_t.

■

Theorem 3 states that there are cases where telling the truth is optimal $(W_t = w_t)$ and there are cases where modifying the real weights is optimal $(W_t \ne w_t)$.

5. SED VS. SID & eBay

In Lemma 4 we proved that $ER^{eBay} \le ER^{SID}$ *assuming the equilibrium specified in Lemma 2.* In this section we try to analyze the sequential protocol vs. the simultaneous and eBay protocols. In Figure 3 and Figure 4 we display the comparison of the expected revenue's values with respect to the three protocols as a function of $\bar{\theta}$ and $\underline{\theta}$, respectively. We use the same parameters' values ($m = 2, n = 5, D = 0.01,$ $\underline{\theta} = 1$, p=0.9, $\bar{\theta} = 10$ $a_1 = a_2 = 1$ $W_1 = W_2 = w_1 = w_2 = 10, reserved = 0$) for Figure 3, 4 and 5 except for the varied parameter.

As illustrated, as the cost of the bidders increases, the expected revenue of the auctioneer in the three protocols decreases. Moreover, we can see that when the bidders are more likely to be homogenous (the relation $\underline{\theta}/\bar{\theta}$ is low), ER^{SED} achieves the best results. And where the bidders are more likely to be heterogeneous (the relation $\underline{\theta}/\bar{\theta}$ is high) ER^{SID} is the best-preferred protocol from the auctioneer's point of view. In the last round of the SED protocol, each bidder sees the bids of the bidders preceding him. Thus, as the bidders are more homogenous each bidder must compete with the bids ahead of him causing higher bids than in the SID protocol, where each bidder behaves according to its beliefs about the other agents. However, if the bidders are heterogeneous, then a strong bidder sees that the bidders preceding him were weak. Thus, he must only consider the bids of the bidders after him. Consequently, with the SED protocol he can offer a bid that is more beneficial to him than in the SID protocol, where he has to consider all the other bidders, since he does not know their strength. Even though the differences seem insignificant, for higher scales they are important.

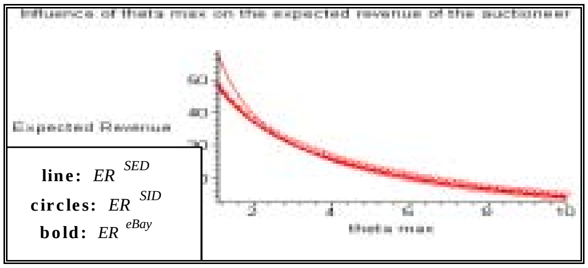

Figure 3. The expected revenue as a function of $\bar{\theta}$.

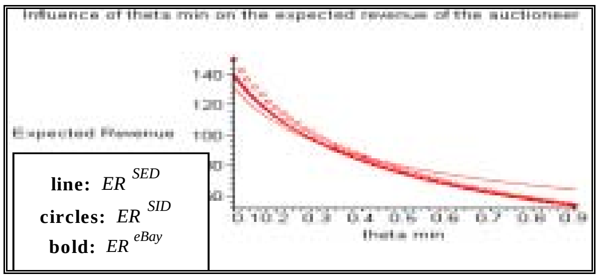

Figure 4. The expected revenue as a function of $\underline{\theta}$.

In Figure 5 the relation between the optimal weights of the scoring function and the real weights of the auctioneer's utility function is demonstrated as a function of $\bar{\theta}$. As $\bar{\theta}$ increases, the w_t / W_t relation decreases. That is, as the bidders are more heterogeneous the auctioneer is motivated to modify the announced weights. Notice that the optimal weights are less or equal to the real weights.

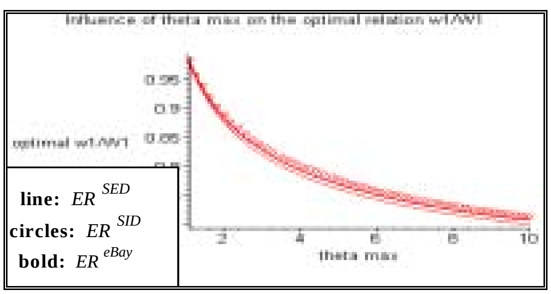

Figure 5. The relation w_t / W_t as a function of $\bar{\theta}$.

6. CONCLUSION

In this paper we provide an auctioneer agent with an optimal auction design in the sequential and simultaneous multi-attribute English-protocols. The main feature of these protocols is that they include a deadline rule that prevents application of the last minute bidding strategy. An additional goal of this paper was to compare the auctioneer agent's results from the sequential and the simultaneous protocols vs. a widespread used auction protocol that includes a deadline rule and in which the bidders use the un-recommended strategy of last minute bidding. We chose the eBay protocol as an example of such protocols because of its popularity. For comparison sake, a full analysis of eBay is provided. In conclusion, we proved that the eBay protocol is dominated by the simultaneous protocol. That is, the auctioneer agent's expected revenue from the eBay protocol is less or equal to the auctioneer agent's expected revenue from the simultaneous protocol. In addition we proved that there are cases where the sequential protocol also achieves better results for the auctioneer than the eBay protocol.

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