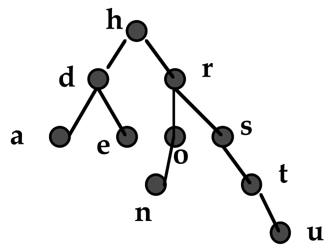
Exam 1

1. [Threaded trees] [30 points]

(a)Draw the symmetric order threads in the following tree (5)



(b) Suppose that you have a binary tree threaded in symmetric order. Explain how you can use the threads in that tree to efficiently locate the **preorder successor** of any node in the tree. **Do not write code - just an English description.** (10)

(c) Illustrate your algorithm using the threaded tree above - make sure you exercise all of the cases in your algorithm. (5)

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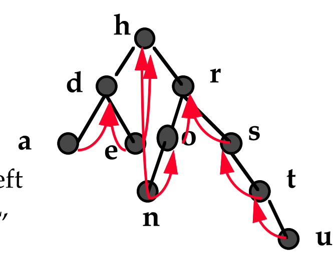


Answer to q1

b

1. If left child is not a thread and is not nil, then left child is pre-order successor (remember: root, L, R) - h, d, r, o
2. Otherwise, if right pointer is not a thread and is not nil, then right child is preorder successor - s,t
3. Right pointer is a thread - follow threads until finding a real pointer, which points to preorder successor (nil means end of chain) - e, n, a

c

1. Suppose node n is left child - if right pointer is thread, it points to parent. Otherwise, follow right pointers until you find a thread - it points to parent (special case at root). Check that left child of this node is n. - d, o, n
2. Symmetric for right

Exam 1

(d) Develop an algorithm to find the **parent** of a node in a threaded tree. Illustrate how your algorithm works using the tree on the previous page. [10]

Exam 1

2. [Storage allocation functions] [25 points]

You are to construct a sequential storage allocation function for **nxn k-banded arrays**. A banded array is an nxn array of integers in which the only nonzero terms occur either on the main diagonal or on one of the k diagonals above and below the main diagonal. So, for example, the following is a picture of a 5x5 1-banded array.

x x 0 0 0
x x x 0 0
0 x x x 0
0 0 x x x
0 0 0 x x

The entries marked "0" always contain 0, while the entries marked "x" might contain nonzero values.

(a) How many nonzero entries can a 10x10 2-banded array have? [5]

(b) How many nonzero entires can an nxn k banded matrix have? [5]

(b) Develop a sequential storage allocation function for the nonzero elements of an nxn k banded array that uses only as much storage as the maximum possible number of nonzero entries (your answer from part 1) Explain how your solution works[15].

Answer to question 2

0	5	0	0	0
9	1	6	0	0
0	10	2	7	0
0	0	11	3	8
0	0	0	12	4

b. Nonzero elements =

$$n + 2\sum_{i=1}^{k}(n-i) = n + 2kn - 2\sum_{i=1}^{k} i = n + 2kn - (k + k^2)$$

c. Storage allocation function:

a) on main diagonal (i=j) A[i,j] = M[i]-1
b) if (i < j) A[i,j] = M[2n-j+i+1)(j-i) -n +i -1]

Question 2

Case i<j

In diagonal (j-i)

skip 2(j-i-1) diagonals

diagonal r has n-r elements

$$M[i,j] = n + 2\sum_{r=1}^{j-i-1}(n-r) + (i-1)$$

$$= n + 2n(j-i-1) - 2\sum_{r=1}^{j-i-1} r + (i-1)$$

$$= n + 2n(j-i-1) - \cancel{2}\frac{(j-i-1)}{\cancel{2}}(j-i\cancel{-1+1}) + (i-1)$$

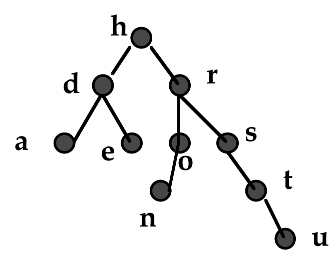
$$= n - 2n + (j-i)2n - (j-i)(j-i-1) + (i-1)$$

$$= (j-i)(2n - j + i + 1) - n + (i-1)$$

Exam 1

3. Binary trees [25 points]

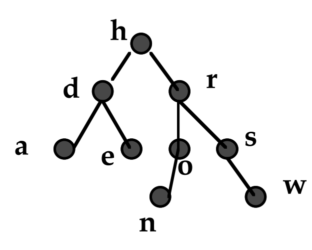
Show the effect of performing a lookup(u) operation on the binary tree given below, treating the tree as a **splay tree**. Show the effects of **each rotation [15**]



Exam 1

3. continued

Show the result of inserting (u) into the following tree regarding the tree as an AVL tree. [10]



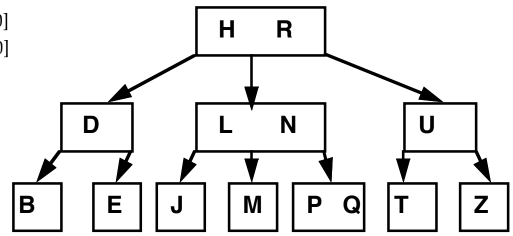
Exam 1

4. Show the results of the following sequence of operations on the 2-3 tree shown below [20 points]

(a) Delete E[10]

(b)Delete H[10]

Show all steps



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