ENEE 324 Assignment #3.

노트 제목 2009-02-05

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1.7.4 1.7.5 1.8.3 1.9.4.

1.7.4

$$\text{Coin A} \begin{cases} P[A_h] = \frac{1}{4} \\ P[A_t] = \frac{3}{4} \end{cases}$$

$$\text{Coin B} \begin{cases} P[B_h] = \frac{3}{4} \\ P[B_t] = \frac{1}{4} \end{cases}$$

① Choose coin Randomly among A and B

② If it is head → suppose that coin is B.

otherwise → A.

not independent events

$$P[C] = P[\text{the head and the coin is } B]$$
$$+ P[\text{the tail and the coin is } A]$$

not independent events.

$$= P[\text{the coin is } B]\, P[\text{the head} \mid \text{the coin is } B]$$
$$+ P[\text{the coin is } A]\, P[\text{the tail} \mid \text{the coin is } A]$$

$$= \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} = \frac{6}{8} = \frac{3}{4}$$

1.7.5. $P[H \text{ virus}] = \frac{1}{5000}$

$P[\text{test is correct}] = 0.99$

① $P[- \mid H] = ?$

independent

$$\Rightarrow \frac{P[- \text{ and } H]}{P[H]} = \frac{P[-] \times P[H]}{\frac{1}{5000}} = \frac{0.01 \times \cancel{\frac{1}{5000}}}{\cancel{\frac{1}{5000}}} = 0.01$$

② $P[H \mid +] = ?$

$P[+ \mid H] = 1 - P[- \mid H] = 0.99.$

$P[+] = P[+ \text{ and } H] + P[+ \text{ and } H^c]$

$P[- \mid H] = P[+ \mid H^c] = 0.01.$

$$P[+] = P[+ \mid H] \times P[H] + P[+ \mid H^c] \times P[H^c]$$

$$= 0.99 \times \frac{1}{5000} + 0.01 \times \frac{4999}{5000}$$

$$P[H \mid +] = \frac{P[+ \text{ and } H]}{P[+]} = \frac{0.99 \times \frac{1}{5000}}{0.99 \times \frac{1}{5000} + 0.01 \times \frac{4999}{5000}}$$

$$= 0.0194.$$

1.8.3. 52 cards and pick up 2 cards randomly

(a) How many outcomes in Sample Space?

Card 1 Card 2.

$52 \text{ case} \times (52-1) \text{ case}$ (in order).

$= 52 \times 51 = 2652$

(b). ① pick one card : 52

② the number of remain cards which have same type with picked card : 3

$n(E_{ST-DS}) = 52 \times 3 = 156$

(c). $P(E_{ST-DS}) = \frac{\cancel{52} \times 3}{\cancel{52} \times 51} = \frac{\cancel{3}}{\cancel{51}} = \frac{1}{17}$

(d). no order case.

$\Rightarrow$ (a)' $\frac{52 \times 51}{2} = 1326$

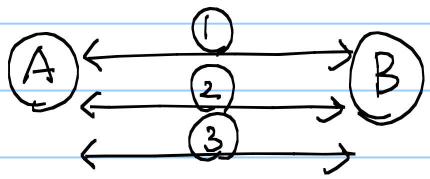
(b)' $\frac{52 \times 3}{2} = 78$

(c)' $\frac{78}{1301} = \frac{3}{51} = \frac{1}{17}$

1.9.4. $\overset{\overset{p}{\shortparallel}}{P[W_h]} > \frac{1}{2}$

$P[L_h] = 1-p.$

$P[W_a] = 1-p \quad P[L_a] = p$



A : start the game with home advantage.

①, ③ : home game

② : away game.

(1) $P[H]$ = Prob[the team which has home advantage win the series]

= P[A win in ① game and A win in ② game]

+ P[A lose in ① game and A win in ② game and A win in ③ game]

+ P[A win in ① and lose in ② and win in ③]

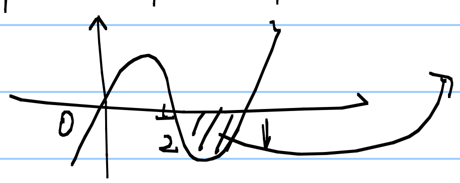
$= p(1-p) + (1-p)^2 p + p^3$

$= p - \cancel{p^2} + \cancel{p^3} - 2\cancel{p^2} + p + \cancel{p^3} = 2p^3 - 3p^2 + 2p.$

(2) $P[H] \geq p$ for all $p \geq \frac{1}{2}$? $\Rightarrow 0 \leq p \leq \frac{1}{2}$

$2p^3 - 3p^2 + 2p - p = f(p) = p(2p^2 - 3p + 1)$

$= p(2p-1)(p-1)$



$f(p) \leq 0$ in this area