ENEE 324

HW #16 Solution

Problem 6.6.2 Solution

Knowing that the probability that voice call occurs is 0.8 and the probability that a data call occurs is 0.2 we can define the random variable D_i as the number of data calls in a single telephone call. It is obvious that for any i there are only two possible values for D_i, namely 0 and 1. Furthermore for all i the D_i's are independent and identically distributed withe the following PMF.

$$P_D(d) = \begin{cases} 0.8 & d = 0 \\ 0.2 & d = 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

From the above we can determine that

$$E[D] = 0.2 \qquad \text{Var}[D] = 0.2 - 0.04 = 0.16 \tag{2}$$

With these facts, we can answer the questions posed by the problem.

(a) $E[K_{100}] = 100E[D] = 20$

(b) $\text{Var}[K_{100}] = \sqrt{100\,\text{Var}[D]} = \sqrt{16} = 4$

(c) $P[K_{100} \geq 18] = 1 - \Phi\left(\frac{18-20}{4}\right) = 1 - \Phi(-1/2) = \Phi(1/2) = 0.6915$

(d) $P[16 \leq K_{100} \leq 24] = \Phi(\frac{24-20}{4}) - \Phi(\frac{16-20}{4}) = \Phi(1) - \Phi(-1) = 2\Phi(1) - 1 = 0.6826$

Problem 6.7.1 Solution

In Problem 6.2.6, we learned that a sum of iid Poisson random variables is a Poisson random variable. Hence W_n is a Poisson random variable with mean $E[W_n] = nE[K] = n$. Thus W_n has variance $\text{Var}[W_n] = n$ and PMF

$$P_{W_n}(w) = \begin{cases} n^w e^{-n}/w! & w = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

All of this implies that we can exactly calculate

$$P[W_n = n] = P_{W_n}(n) = n^n e^{-n}/n! \tag{2}$$

Since we can perform the exact calculation, using a central limit theorem may seem silly; however for large n, calculating n^n or $n!$ is difficult for large n. Moreover, it's interesting to see how good the approximation is. In this case, the approximation is

$$P[W_n = n] = P[n \leq W_n \leq n] \approx \Phi\left(\frac{n + 0.5 - n}{\sqrt{n}}\right) - \Phi\left(\frac{n - 0.5 - n}{\sqrt{n}}\right) = 2\Phi\left(\frac{1}{2\sqrt{n}}\right) - 1 \tag{3}$$

The comparison of the exact calculation and the approximation are given in the following table.

$P[W_n = n]$	$n = 1$	$n = 4$	$n = 16$	$n = 64$
exact	0.3679	0.1954	0.0992	0.0498
approximate	0.3829	0.1974	0.0995	0.0498

(4)