Problem 1.10.2 Solution

Suppose that the transmitted bit was a 1. We can view each repeated transmission as an independent trial. We call each repeated bit the receiver decodes as 1 a success. Using $S_{k,5}$ to denote the event of k successes in the five trials, then the probability k 1's are decoded at the receiver is

$$P[S_{k,5}] = \binom{5}{k} p^k (1-p)^{5-k}, \quad k = 0, 1, \ldots, 5. \tag{1}$$

The probability a bit is decoded correctly is

$$P[C] = P[S_{5,5}] + P[S_{4,5}] = p^5 + 5p^4(1-p) = 0.91854. \tag{2}$$

The probability a deletion occurs is

$$P[D] = P[S_{3,5}] + P[S_{2,5}] = 10p^3(1-p)^2 + 10p^2(1-p)^3 = 0.081. \tag{3}$$

The probability of an error is

$$P[E] = P[S_{1,5}] + P[S_{0,5}] = 5p(1-p)^4 + (1-p)^5 = 0.00046. \tag{4}$$

Note that if a 0 is transmitted, then 0 is sent five times and we call decoding a 0 a success. You should convince yourself that this a symmetric situation with the same deletion and error probabilities. Introducing deletions reduces the probability of an error by roughly a factor of 20. However, the probability of successfull decoding is also reduced.

Problem 2.2.4 Solution

(a) We choose c so that the PMF sums to one.

$$\sum_x P_X(x) = \frac{c}{2} + \frac{c}{4} + \frac{c}{8} = \frac{7c}{8} = 1 \tag{1}$$

Thus $c = 8/7$.

(b)

$$P[X = 4] = P_X(4) = \frac{8}{7 \cdot 4} = \frac{2}{7} \tag{2}$$

(c)

$$P[X < 4] = P_X(2) = \frac{8}{7 \cdot 2} = \frac{4}{7} \tag{3}$$

(d)

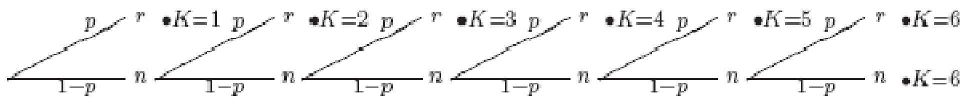
$$P[3 \le X \le 9] = P_X(4) + P_X(8) = \frac{8}{7 \cdot 4} + \frac{8}{7 \cdot 8} = \frac{3}{7} \tag{4}$$

Problem 2.2.6 Solution

The probability that a caller fails to get through in three tries is $(1-p)^3$. To be sure that at least 95% of all callers get through, we need $(1-p)^3 \leq 0.05$. This implies $p = 0.6316$.

Problem 2.2.9 Solution

(a) In the setup of a mobile call, the phone will send the "SETUP" message up to six times. Each time the setup message is sent, we have a Bernoulli trial with success probability p. Of course, the phone stops trying as soon as there is a success. Using r to denote a successful response, and n a non-response, the sample tree is



(b) We can write the PMF of K, the number of "SETUP" messages sent as

$$P_K(k) = \begin{cases} (1-p)^{k-1}p & k = 1, 2, \ldots, 5 \\ (1-p)^5 p + (1-p)^6 = (1-p)^5 & k = 6 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Note that the expression for $P_K(6)$ is different because $K = 6$ if either there was a success or a failure on the sixth attempt. In fact, $K = 6$ whenever there were failures on the first five attempts which is why $P_K(6)$ simplifies to $(1-p)^5$.

(c) Let B denote the event that a busy signal is given after six failed setup attempts. The probability of six consecutive failures is $P[B] = (1-p)^6$.

(d) To be sure that $P[B] \leq 0.02$, we need $p \geq 1 - (0.02)^{1/6} = 0.479$.