ENEE 324 HW 9

노트 제목 2009-03-10

Homework #9: 3.7.11, 3.8.4, 4.1.5, and 4.2.7 (due Thursday March 12).

by inkenny

3.7.11.

U. uniform dist $[-1, 1]$.

$$W = g(U) = \begin{cases} 0 & U < 0 \\ U & U \geq 0 \end{cases}$$

$F_W(w)$, $E[W] = ?$

$F_W(w) = P[W \leq w]$

$\Rightarrow P[W = 0] = P[-1 \leq U \leq 0] = \int_{-1}^{0} \frac{1}{2}\, du = \frac{1}{2}$

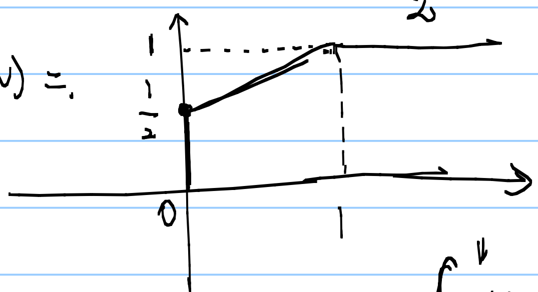
$\Rightarrow P[W \leq w] = P[W = 0] + P[0 < W \leq w]$

$= P[W = 0] + P[0 < U < u]$

$= \frac{1}{2} + \frac{1}{2} u$

$\int_0^u \frac{1}{2}\, du = \frac{1}{2} u$

$F_W(w) =$



$(F_W(w))'$ $\Rightarrow$ $\frac{1}{2}$

$E[W] = 0 \times P[W = 0] + \int_0^1 w' \cdot f_W(w')\, dw'$

$= 0 + \left[\frac{1}{4} w'^2\right]_0^1 = \frac{1}{4}$

3.8.4. $W \sim \mathcal{N}(\mu = 0,\ \sigma^2 = 16$ Gaussian R.V).

$C = \{W > 0\}$.

(a). $f_{W|C}(w)$?

$\Rightarrow P[W = w' \mid W > 0] = X$.

$$\Rightarrow \begin{cases} w' > 0 \Rightarrow X > 0. \\ w' < 0 \Rightarrow X = 0 \end{cases}$$

$$\Rightarrow \frac{P[W = w']}{P[W > 0]} \Rightarrow = \begin{cases} \dfrac{\frac{1}{\sqrt{32\pi}} e^{-\frac{w'^2}{32}}}{\frac{1}{2}} & w' > 0. \\ 0 & w' < 0. \end{cases}$$

(b). $E[W \mid C] = \int_0^\infty w' \, 2 \times \frac{1}{\sqrt{32\pi}} e^{-\frac{w'^2}{32}} dw'$

$$= 2\int_0^\infty w' \times \frac{1}{\sqrt{32\pi}} e^{-\frac{w'^2}{32}} dw'$$

$\frac{w^2}{32} = v$

$\frac{1}{16} w\,dw = dv$,

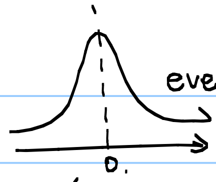
$w\,dw = 16\,dv$

$$= 2\int_0^\infty \frac{1}{\sqrt{32\pi}} 16 e^{-v} dv.$$

$$= \frac{\sqrt{32}}{\pi} \int_0^\infty e^{-v} dv = \frac{\sqrt{32}}{\pi}$$

(c). $Var[W|C]$

even fnc.



$$\Rightarrow \; E[W^2|C] = 2\int_0^{\infty} w^2 f_W(w)\, dw.$$

$$= \int_{-\infty}^{\infty} w^2 f_W(w)\, dw$$

$$= \int_{-\infty}^{\infty} w^2 f_W(w) - \underset{\substack{\| \\ (E[W])^2}}{0^2} f_W(w)^2 \, dw$$

$$= Var[W]$$

$$= 16.$$

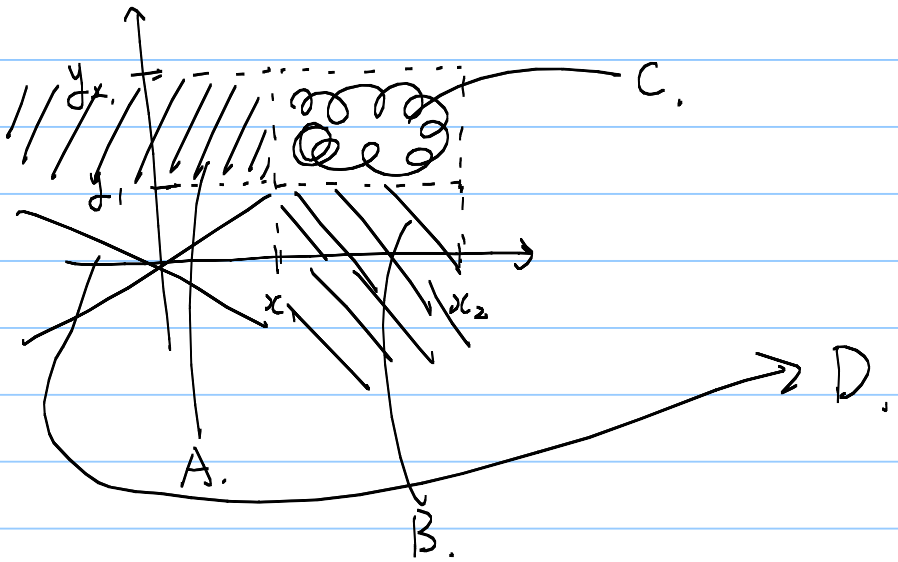
$$\therefore \; Var[W|C] = E[W^2|C] - \{E[W|C]\}^2$$

$$= 16 - \frac{32}{\pi} = 5.81.$$

4.1.5.

(a) $A = \{ X \le x_1, \ y_1 \le Y \le y_2 \}$. $B = \{ x_1 \le X \le x_2, \quad Y \le y_1 \}$.

$C = \{ x_1 \le X \le x_2, y_1 \le Y \le y_2 \}$.



(b) $P[A] = F_{X,Y}(x_1, y_2) - F_{X,Y}(x_1, y_1)$.

$P[B] = F_{X,Y}(x_2, y_1) - F_{X,Y}(x_1, y_1)$.

$$\boxed{P[C] = F_{X,Y}(x_2, y_2) - F(x_1, y_2) - F(x_2, y_1) + F_{X,Y}(x_1, y_1).}$$

$P[A \cup B \cup C] = F_{X,Y}(x_2, y_2) - F_{X,Y}(x_1, y_1)$

(c). (c) Since A, B, and C are mutually exclusive,

$$P[A \cup B \cup C] = P[A] + P[B] + P[C]$$

However, since we want to express

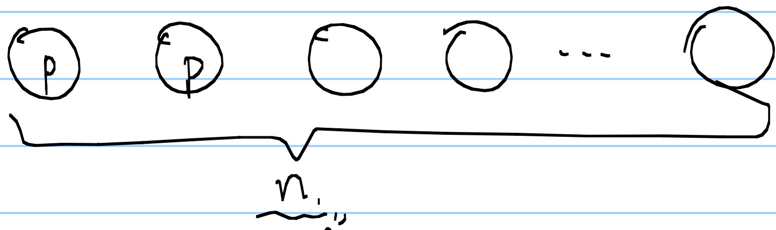
$$P[C] = P[x_1 < X \le x_2, y_1 < Y \le y_2]$$

in terms of the joint CDF $F_{X,Y}(x, y)$, we write

$$\begin{aligned} P[C] &= P[A \cup B \cup C] - P[A] - P[B] \\ &= F_{X,Y}(x_2, y_2) - F_{X,Y}(x_1, y_2) - F_{X,Y}(x_2, y_1) + F_{X,Y}(x_1, y_1) \end{aligned}$$

which completes the proof of the theorem.

4.2.7.



k: the number of circuits rejected.

x: the number of acceptable circuit in the last test.

$P_{K,X}(k,x)$.

k−1 rejected circuits among n−1 circuits, rejected.

$$\Rightarrow \begin{cases} P_{K,X}(k,0) \Rightarrow \underbrace{\ominus\ \ominus\ \cdots}_{k-1 \text{ rejected circuits among } n-1 \text{ circuits}}\ 0 \ (\text{rejected}) \\ P_{K,X}(k,1) \Rightarrow \underbrace{\ominus\ \ominus\ \cdots}_{k \text{ rejected circuits among } n-1 \text{ circuits}}\ 0 \ (\text{accepted}) \end{cases}$$

$$\Rightarrow \begin{cases} P_{K,X}(k,0) = \binom{n-1}{k-1}(1-p)^{k-1}p^{(n-k)} \times (1-p) & 1 \le k \le n \\ P_{K,X}(k,1) = \binom{n-1}{k}(1-p)^{k}p^{(n-k-1)} \times p & 0 \le k \le n \end{cases}$$

$$\Rightarrow P_{K,X}(k,x) = \begin{cases} \binom{n-1}{k-1}(1-p)^{k}p^{(n-k)} & 1 \le k \le n, x=0 \\ \binom{n-1}{k}(1-p)^{k}p^{(n-k)} & 0 \le k \le n, x=1 \\ 0 & \text{otherwise} \end{cases}$$