# ENEE 324 HW 7

노트 제목 2009-02-19

Homework #7: 3.2.4, 3.2.5, 3.3.4, and 3.3.6

by inkeun.

3.2.4 Rayleigh R.V. p.d.f.

$$f_X(x) = \begin{cases} a^2 x e^{-\frac{a^2x^2}{2}} & x>0, \\ 0 & \text{o.w.} \end{cases}$$

$F_X(x) = ?$

$$F_X(x) = \int_0^x f_T(t)\,dt,$$

$$= \int_0^x a^2 t e^{-\frac{a^2t^2}{2}}\,dt,$$

$$= \left[ -e^{-\frac{a^2t^2}{2}} \right]_0^x$$

$$= 1 - e^{-\frac{a^2x^2}{2}} \quad x \geq 0.$$

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ 1-e^{-\frac{a^2x^2}{2}} & x>0 \end{cases}$$

3.2.5.

$$f_X(x) = \begin{cases} ax^2+bx & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

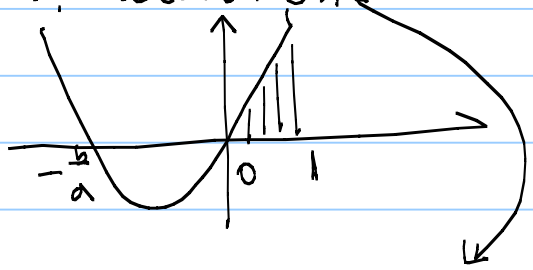
i) $\int_0^1 f_X(x) = 1$.

ii) $f_X(x) \ge 0$.

i) $\Rightarrow \int_0^1 ax^2+bx\,dx = \left[\frac{1}{3}ax^3+\frac{1}{2}bx^2\right]_0^1 = \frac{1}{3}a+\frac{1}{2}b = 1$

$2a+3b=6$. —①. $\quad b = -\frac{2}{3}a+2$

ii) $\Rightarrow$ $x(ax+b)$.




$-\frac{b}{a} < 0$.

$\frac{b}{a} > 0$,

$\Rightarrow x\left(ax-\frac{2}{3}a+2\right) \ge 0$. $(\because f(x) \ge 0)$.

$x \ge 0$.

$ax-\frac{2}{3}a+2 \ge 0$ for $0 \le x \le 1$.

$K = a\left(\frac{2}{3}-x\right) \le 2$

we should consider two condition.

① $0 \le x \le \frac{2}{3}$ ② $\frac{2}{3} \le x \le 1$.

① $\rightarrow \frac{2}{3}-x \le 0$.

② $\rightarrow \frac{2}{3}-x \ge 0$.

in ① condition, in case $x=0 \Rightarrow K$ is the largest. $\Rightarrow \frac{2}{3}a$.

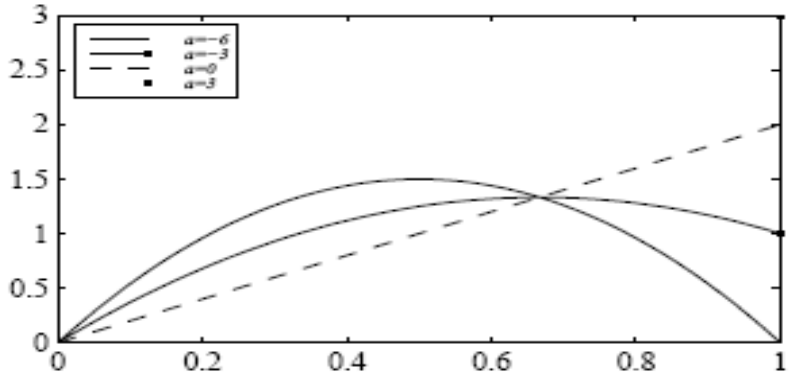
$\frac{2}{3}a \le 2 \quad a \le 3$.

in ② condition, in case. $x=1 \Rightarrow k$ is the smallest

$-\frac{1}{3}a \leq 2$. $\quad a \geq -6$.

so, $-6 \leq a \leq 3$, and, $b = -\frac{2}{3}a + 2$.

the pdf value varies with a and b.




3.3.4. $f_Y(y) = \begin{cases} \frac{y}{2} & 0 \leq y \leq 2 \\ 0 & \text{o.w.} \end{cases}$

(i) $E[Y] = ?$

$$E[Y] = \int_0^2 y \times \frac{y}{2}\, dy = \int_0^2 \frac{y^2}{2}\, dy = \left[\frac{1}{6}y^3\right]_0^2 = \frac{8}{6} = \frac{4}{3}$$

(ii) $Var[Y] = ?$

$$Var[Y] = E[Y^2] - \{E[Y]\}^2$$

$$E[Y^2] = \int_0^2 y^2 \times \frac{y}{2}\, dy = \int_0^2 \frac{y^3}{2}\, dy = \left[\frac{1}{8}y^4\right]_0^2 = 2$$

$$Var[Y] = 2 - \frac{16}{9} = \frac{2}{9}$$

3.3.6.

$$F_V(v) = \begin{cases} 0 & v < -5 \\ (v+5)^2/144 & -5 \le v < 7 \\ 1 & v \ge 7. \end{cases}$$

(a) What is $E[V]$?

$$f_V(v) = \begin{cases} 0 & v < -5 \\ \frac{(v+5)}{72} & -5 \le v < 7. \\ 0 & v \ge 7. \end{cases}$$

$$E[V] = \int_{-5}^{7} v\, f_V(v)\, dv = \int_{-5}^{7} \frac{v(v+5)}{72}\, dv = \int_{-5}^{7} \frac{v^2}{72} + \frac{5}{72} v\, dv$$

$$= \left[ \frac{1}{216} v^3 + \frac{5}{144} v^2 \right]_{-5}^{7}$$

$$= \frac{1}{72}\left( \frac{343}{3} + \frac{245}{2} + \frac{125}{3} - \frac{125}{2} \right) = 3$$

ii) $E[V^2] = \int_{-5}^{7} \frac{v^3 + 5v^2}{72}\, dv$

$$= \frac{1}{72}\left[ \frac{v^2}{4} + \frac{5v^3}{3} \right]_{-5}^{7} = \frac{6719}{432} = 15.55.$$

$$\mathrm{Var}[V] = E[V^2] - (E[V])^2 = \frac{2831}{432} = 6.55$$

iii) $E[V^3] = \frac{1}{72} \int_{-5}^{7} (v^4 + 5v^3)\, dv$

$$= \frac{1}{72}\left[ \frac{v^5}{5} + \frac{5v^4}{4} \right]_{-5}^{7} = 86.2$$