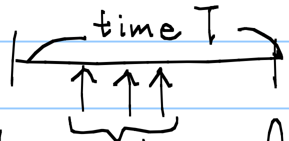
ENEE 324 HW 5

노트 제목 2009-02-12

Homework5: 2.3.7, 2.3.10, 2.5.10, and 2.5.11

–by inkeun

2.3.7.



the number of buses that arrive at a bus stop : B.

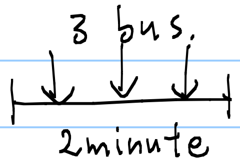
$E[B] = \frac{T}{5}$ and Poisson distributed.

(a) P.M.F of B ?

$\alpha = \frac{T}{5}$ in this case.

$$P[B=n] = \frac{\alpha^n}{n!} e^{-\alpha} \Rightarrow \frac{\left(\frac{T}{5}\right)^n}{n!} e^{-\alpha}.$$

(b)



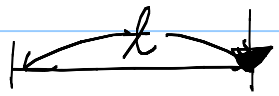
$T=2 \quad n=3.$

$$P[B=3] = \frac{\left(\frac{2}{5}\right)^3}{3!} e^{-\frac{2}{5}} \approx 0.0072$$

(c) No bus within 10 minute.

$$P[B=0] = \frac{\left(\frac{10}{5}\right)^0}{0!} e^{-\frac{10}{5}} \approx 0.135$$

(d)



$$P[B \geqq 1 \text{ within } T] = 1 - P[B=0 \text{ within } T]$$
$$= 1 - \frac{\left(\frac{T}{5}\right)^0}{0!} e^{-\frac{T}{5}}$$
$$= 1 - e^{-\frac{T}{5}} \geqq 0.99,$$
$$e^{-\frac{T}{5}} \leqq 0.01$$
$$-\frac{T}{5} \leqq \ln 0.01,$$
$$T \geqq -5 \ln 0.01 \approx 23.$$

2.3.10.

○ R.S.

○ sixth caller who knows the birthday of performer

the probability that caller know perfomer's B.D
$= P[C_N] = 0.75.$

all calls are independent.

(a) L: the number of call

$$P[L=n] = \begin{cases} 0 & n=0,1,\cdots 5, \\ (0{,}75)^6 & n=6. \\ \binom{6}{5}(0{,}75)^5\, 0{,}25 \times 0{,}75 & n=7. \\ \binom{k-1}{5}(0{,}75)^5\, 0{,}25^{(k-6)} \times 0{,}75 & \end{cases}$$

$$\Rightarrow P[L=n] = \begin{cases} 0 & n=0,1,\cdots 5 \\ \binom{n-1}{5}(0{,}75)^6\, 0{,}25^{(n-6)} & n \geqq 6 \end{cases}$$

(b) $P[L=10] = \binom{9}{5}(0{,}75)^6\, 0{,}25^4 \approx 0{,}0876.$

(c) $P[L \geqq 9] = 1 - P[L=6] - P[L=7] - P[L=8]$

$$= 1 - (0{,}75)^6 - (0{,}75)^6 \binom{6}{5} 0{,}25$$

$$- (0{,}75)^6 \binom{7}{5} 0{,}25^2$$

$$\approx 0{,}321$$

2.5.10

$$E[X_n] = \sum_{x=1}^{n} x \binom{n}{x} p^{x} (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x \frac{n!}{x!(n-x)!} p^{x}(1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^{x}(1-p)^{n-x}$$

$$= \sum_{x=1}^{n} np \frac{(n-1)!}{(x-1)!(n-1-(x-1))!} p^{x-1}(1-p)^{(n-1-(x-1))}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-1-(x-1))!} p^{x-1}(1-p)^{(n-1-(x-1))}$$

$\Rightarrow x-1 = x'$

$$= np \sum_{x'=0}^{n-1} \frac{(n-1)!}{x'!(n-1-x')!} p^{x'}(1-p)^{(n-1-x')}$$

$$= np \underbrace{\sum_{x'=0}^{n-1} \binom{n-1}{x'} p^{x'}(1-p)^{(n-1-x')}}_{1}$$

(∵ the sum of binomial dist pmf is 1)

$$= np$$

2, 5, 11.

$$E[X] = \sum_{k=0}^{\infty} P[X > k].$$

$$E[X] = \sum_{k=0}^{\infty} k P[X = k]$$

$$= \sum_{k=0}^{\infty} \underbrace{P[X = k+1]}_{①} + \underbrace{P[X = k+2]}_{②} + \underbrace{P[X = k+3]}_{③} + \cdots$$

$$= \underbrace{P[X = 1] + P[X=2] + P[X=3] \cdots}_{①}$$

$$\underbrace{+ P[X=2] + P[X=3] \cdots}_{②}$$

$$\underbrace{+ P[X=3] + \cdots}_{③}$$

$$= \sum_{k=0}^{\infty} k P[X = k]$$