HW 17

노트 제목 2009-04-27

7.1.4 7.2.2 7.2.4.

7.1.4 X_i : indep. with $Var[X]$.

$Y_1 = X_1 - X_2$.
$Y_2 = X_3 - X_4$,
$Y_3 = X_5 - X_6$.
$Y_n = X_{2n-1} - X_{2n}$.

(a). $E[Y_n]$?

independent sample of X, $\Downarrow$ $E[X]$.

$$E[Y_n] = E[X_{2n-1}] - E[X_{2n}].$$
$$= E[X] - E[X] = 0.$$

(b). $Var[Y_n] = Var[X_{2n-1} - X_{2n}]$.

by theorem 6.2.

$$= Var[X_{2n-1}] + Var[-X_{2n}]$$
$$= 2 Var[X].$$

(c) $M_n(Y)$, $E[M_n(Y)]$, $Var[M_n(Y)]$

$$E[M_n(Y)] = E[Y] = 0,$$
$$Var[M_n(Y)] = \frac{Var[Y]}{n} = \frac{2Var[X]}{n}$$

7.2.2.

$$P[|X-E[X]| \geq k\sigma] \leq \frac{1}{k^2}$$

the chevishev inequality.

$$P[|X-E[X]| \geq c] \leq \frac{\sigma_x^2}{c^2}$$

if we choose $c = k\sigma_x$,

$$P[|X-E[X]| \geq k\sigma] \leq \frac{1}{k^2}$$

For Gaussian r.v Y,

$$P[|Y-E[Y]| \geq k\sigma_Y]$$

$$= P[Y-E[Y] \leq -k\sigma_Y] + P[Y-E[Y] \geq k\sigma_Y]$$

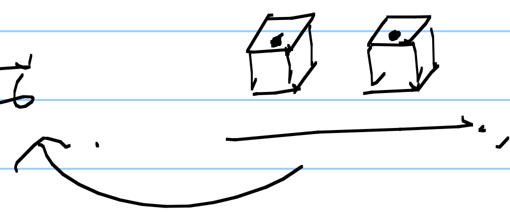
$$= 2P\left[\frac{Y-E[Y]}{\sigma_Y} \geq k\right]$$

$$= 2Q(k).$$

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
Chevy bound	1	0.25	0.111	0.0625	0.040
$2Q(k)$.	0.317	0.046	0.0027	6.3×10^{-5}	5.37×10^{-7}.

7.2.4.

$$P_{success} = \frac{1}{36}$$



R: the number of trial to get three success.

$\Rightarrow$ Pascal $(k=3, p)$ random variable.

$$E[R] = \frac{3}{p} = 108 \qquad Var[R] = \frac{3(1-p)}{p^2} = 3780.$$

(a) $$P[R \geq 250] \leq \frac{E[R]}{250} = \frac{54}{125} = 0.432.$$

(b). $$P[R \geq 250] = P[R - 108 \geq 142]$$

$$= P[|R-108| \geq 142] \leq \frac{Var[R]}{(142)^2} = 0.1875$$

(c). $$P[R \geq 250] = 1 - \sum_{r=3}^{249} P_R(r).$$

$$= 0.0299.$$

Markov and Chevychev inequality give us upper bound, however it is not a good estimate.