HW 14.

노트 제목 2009-04-21

6.1.3, 6.1.5 6.2.3 6.2.4,

6.1.3 4th caller get a ticket

$P[\text{Right}] = 0.25$.

N_r: number of phone calls taken, when the rth correct answer arrives

(a) $P_{N_1}[r] = \begin{cases} (0.75)^{r-1}(0.25) & r = 1, \cdots \infty \\ 0, & r = 0. \end{cases}$

(b). $E[N_1] = \sum_{r=1}^{\infty} r \left(\frac{3}{4}\right)^{r-1}(0.25)$ ⇒ geometric r.v with $p = \frac{1}{4}$.

$= \dfrac{1}{\frac{1}{4}} = 4$.

(c) 3 right answers during n-1 call

$\underbrace{0 \cdots}\ \underbrace{0\ 0\ 0\ 0}\ 0$ ↙ correct,

$$P_{N_4}[n] = \begin{cases} \binom{n-1}{3}\left(\frac{3}{4}\right)^{n-4}\left(\frac{1}{4}\right)^4 & n = 4, 5, \cdots \\ 0 & \text{o.w} \end{cases}$$

(d). $E[N_4] = 4E[N_1] = 16.$

the mean N_4 can be acquired by summing up the mean of the 4 identically distributed geometric r.v. each with mean 4.

6.1.5.
$$C_X[m,k] = \begin{cases} 1 & k=0 \\ \frac{1}{4} & |k|=1 \\ 0 & \text{o.w.} \end{cases}$$

$$X \longrightarrow \text{filter} \xrightarrow{Y}$$

$$Y_n = \frac{X_n + X_{n-1} + X_{n-2}}{3}$$

$$C_X[m,k] = E[(X_m - E(X_m))(X_k - E(X_k))]$$
$$= E[X_m X_k] - E[X_m]E[X_k]$$

$$C_X[m,0] = E[X_m X_m] - E[X_m]E[X_m] = 1.$$
$$C_X[m,-1] = E[X_m X_{m-1}] - E[X_m]E[X_{m-1}] = \frac{1}{4}$$
$$C_X[m,1] = E[X_m X_{m+1}] - \underbrace{E[X_m]}_{=0}E[X_{m+1}] = \frac{1}{4}$$

$$E[X_m^2] = 1 \quad E[X_m X_{m-1}] = E[X_m X_{m+1}] = \frac{1}{4}$$

$$E[Y_n] = \frac{1}{3}E[X_n] + \frac{1}{3}E[X_{n-1}] + \frac{1}{3}E[X_{n-2}]$$

$$= 0$$

$$E[Y_n^2] = E\left[\frac{X_n^2 + X_{n-1}^2 + X_{n-2}^2 + 2X_nX_{n-1} + 2X_nX_{n-2} + 2X_{n-1}X_{n-2}}{9}\right]$$

$$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}\left(2\times\frac{1}{4} + 2\times\frac{1}{4} + 2\times 0\right)$$

$$= \frac{1}{3} + \frac{1}{9} = \frac{4}{9}$$

$$Var[Y_n] = \frac{4}{9}$$

6.2.3. X, Y : indep exponential r.v with

$E[X] = \frac{1}{\lambda}$ $E[Y] = \frac{1}{\mu}$

$W = X + Y$. ① $\lambda \neq \mu$ ② $\lambda = \mu$.

② $f_W(w) = f_{X+Y}(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(w-x)\,dx$

$$= \int_0^w \lambda e^{-\lambda x} \lambda e^{-\lambda(w-x)}\,dx$$

$$= \lambda^2 e^{-\lambda w} \int_0^w dx$$

$$= \begin{cases} \lambda^2 w e^{-\lambda w} & w \geq 0. \\ 0 & \text{o.w.} \end{cases}$$

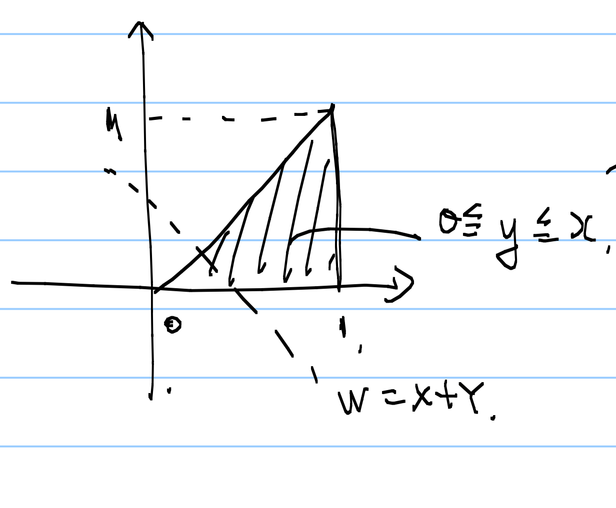
①. $f_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(w-x) dx$

$$= \int_0^w \lambda e^{-\lambda x} \mu e^{-\mu(w-x)} dx$$

$$= \lambda \mu e^{-\mu w} \int_0^w e^{-(\lambda-\mu)x} dx$$

$$= \begin{cases} \frac{\lambda\mu}{\lambda-\mu} (e^{-\mu w} - e^{-\lambda w}) & w \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

6.2.4. $f_{X,Y}(x,y) = \begin{cases} 8xy & 0 \leq y \leq x \leq 1 \\ 0 & \text{o.w.} \end{cases}$



$W = X+Y$

$0 \leqq w \leqq 1$

$y = w - x,$

$$f_W(w) = \int_{\frac{w}{2}}^{w} 8x(w-x)\,dx$$

$$= 4wx^2 - \frac{8x^3}{3}\Big]_{\frac{w}{2}}^{w} = \frac{2w^3}{3}$$

$1 \leqq w \leqq 2,$

$$f_W(w) = \int_{\frac{w}{2}}^{1} 8x(w-x)\,dx$$

$$= 4wx^2 - \frac{8x^3}{3}\Big]_{\frac{w}{2}}^{1}$$

$$= 4w - \frac{8}{3} - \frac{2w^3}{3}$$

o.w $= 0.$

$$f_W(w) = \begin{cases} \frac{2w^3}{3} & 0 \leqq w \leqq 1 \\ 4w - \frac{8}{3} - \frac{2w^3}{3} & 1 \leqq w \leqq 2 \\ 0 & \text{o.w.} \end{cases}$$