Phil 370
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Sets, relations, functions

1. Some definitions and facts:

$$A \subseteq B =_{df} \forall x\,(x \in A \rightarrow x \in B)$$

$$A = B =_{df} A \subseteq B \wedge B \subseteq A$$

$$A \subset B =_{df} A \subseteq B \wedge \sim(B \subseteq A)$$

$$A \cup B =_{df} \{x : x \in A \vee x \in B\}$$

$$A \cap B =_{df} \{x : x \in A \wedge x \in B\}$$

$$A - B =_{df} \{x : x \in A \wedge x \notin B\}$$

$\emptyset$ governed by axiom: $\sim\exists x\,(x \in \emptyset)$

U governed by axiom: $\forall x\,(x \in U)$

$$\mathcal{P}(A) =_{df} \{B : B \subseteq A\}$$

$$\{a_1, a_2, \ldots a_n\} =_{df} \{x : x = a_1 \vee x = a_2 \vee \ldots\ x = a_n\}$$

2. Naive abstraction (and unabstraction):

$$\frac{A^a/x}{a \in \{x : A\}} \in I$$

$$\frac{a \in \{x : A\}}{A^a/x} \in E.$$

3. Cartesian products, relations:

$X \times Y = \{\langle a, b\rangle : a \in X \wedge b \in Y\}$

$X_1 \times X_2 \times \dots X_n = \{\langle a_1, \dots, a_n\rangle : a_i \in X_i\}$

A relation R is a subset of an n-ary Cartesian product.

Eg: $R \subseteq X \times Y$. Here, X is the domain and Y the range of the relation.

4. Functions

A function $f: X \to Y$ is a relation $f \subseteq X \times Y$ subject to condition that

$$\forall a, b, c \left[(\langle a, b\rangle \in f \wedge \langle a, c\rangle \in f) \to b = c \right].$$

We say that: $f(a) = b$