Philosophy 470: Logical Theory I

Description

This course has two goals. The primary goal is to give students a thorough understanding of the proof theory and semantics of classical propositional and first-order logic, as well as the rudiments of set theory; we prove fundamental metatheoretic results, such as soundness, completeness, and compactness. The secondary goal of the course is to provide an overview of some of the most important alternatives to and extensions of classical logic, such as intuitionistic or modal logics.

Time and place

The class meets Thursdays from 2:00 till 4:30 in ARM 0110.

Office, phones, etc.

Office: AV Williams 3133, and also somewhere in Skinner Hall. Office phone: x56749. Home phone: 301-585-8714. You are welcome to call me at home, but not after 9 pm, please. I’ll announce office hours once they sort themselves out. The most likely time is Tuesday afternoons, but I’m not sure yet which office.

Course work

There will be regular assigned exercises, and at least two exams, which may have the form of take-home problem sets. Students and auditors will be asked to help with official note-taking, and may also be asked to give short presentations of straightforward material.

Materials

Many of the topics to be covered are treated in the following books, available in the bookstore:


In addition, I’ll be handing out lots of selections from various books and articles.

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1 Please ignore the fact that the official number is temporarily 688L.
Topics

1. Background
   (a) Informal set theory
   (b) Induction

2. Classical propositional logic
   (a) Fitch-style deduction formulations
   (b) Hilbert-style axiomatic formulations
   (c) Equivalence of formulations
      i. Deduction theorem
      ii. Equivalence of F-style and H-style formulations
   (d) Resolution
      i. CNF and clauses
      ii. Resolution rule
   (e) Semantics for classical logic
      i. Valuation
      ii. Satisfaction
      iii. Satisfiability
      iv. Simultaneous satisfiability
      v. Semantic implication
      vi. Validity
   (f) Soundness, completeness
      i. Soundness theorem
      ii. Completeness theorem (Henkin)
      iii. Completeness theorem (Kalmar)
      iv. Compactness theorem
      v. Interpolation theorem
      vi. Decidability

3. First-order logic
   (a) Fitch-style formulations
   (b) Hilbert-style formulations
   (c) Equivalence of formulations
      i. Deduction theorem
      ii. Equivalence of F-style and H-style formulations
   (d) Resolution
      i. Unification and mgu’s
      ii. Resolution with unification
   (e) Semantics for first-order logic
      i. Model (domain and valuation)
      ii. Satisfaction ——
iii. Satisfiability
iv. Simultaneous satisfiability
v. Semantic implication ———
vi. Validity

(f) Soundness, completeness
   i. Soundness theorem
   ii. Completeness theorem (Henkin)
   iii. Compactness theorem
   iv. Interpolation theorem

(g) First order logic with identity
   i. Fitch and Frege style formulations
   ii. Semantics of identity
   iii. Modifications of completeness proof
   iv. Elimination of function symbols and constants

(h) Some elementary model theory
   i. Isomorphic models, categoricity
   ii. Lowenheim-Skolem theorem

4. Definitions and theories

   (a) Classical account of definitions
      i. Eliminability and conservativeness
      ii. Rules for definition
      iii. Beth’s definability theorem

   (b) Formalizing some theories
      i. Number theory
      ii. Set theory

5. Some nonclassical logics

   (a) Many-valued logics
   (b) Modal logics (K,T,B,S4,S5)
      i. Possible worlds semantics
      ii. Fitch-style formulations
      iii. Hilbert-style formulations
      iv. Soundness
      v. Completeness (Henkin style)
   (c) Intuitionistic logic
      i. Fitch-style formulations
      ii. Hilbert-style formulations
      iii. Double-negation interpretations into classical logic
      iv. Interpretations into S4
      v. Kripke semantics