Bounded Stretch Homotopic Routing Using Hyperbolic Embedding of Sensor Networks

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In this paper we consider lightweight routing in a wireless sensor network deployed in a complex geometric domain Σ with holes. Our goal is to find short paths of different *homotopy types*, i.e., paths that go around holes in different ways. In the example of Figure 1, there are three holes in the network and there are many different ways to "thread" a route from s to t. Observe that paths α, β, γ are all different in a global sense; in that, e.g., one cannot deform α to β without "lifting" it over some hole. In contrast, paths γ and δ are only different in a local manner; one can deform γ to δ continuously through local changes, keeping δ within the domain. This difference is characterized by the *homotopy type* of a path. Two paths in a Euclidean domain are *homotopy equivalent* if one can continuously deform one to the other. A set of paths that are pairwise homotopy equivalent are said to have the same homotopy type. The number of homotopy types is infinitely many (assuming there is at least one hole), as a path can loop around a hole k times, for any integer k; however, for most routing scenarios we only care about a finite number of homotopy types, corresponding to paths in the dual graph of a triangulation of Σ that do not repeat triangles (and thus do not loop around holes).

One heuristic algorithm gives a heuristic path for a given case. The ratio between the length of the heuristic path and the length of the optimal path is defined as the *stretch* of this algorithm for the case.

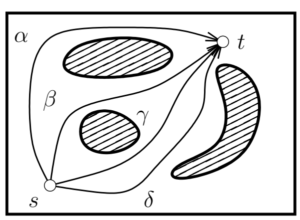

Fig. 1. The network has 3 holes (shaded). Paths α, β, γ have distinct homotopy types; γ and δ are homotopy equivalent.

We introduce a routing framework that guarantees constant worst case stretch for a given homotopy type. We assume that the network is deployed in a geometric domain that is represented by a polygon Σ. We decompose the domain Σ into a triangulation, by including certain diagonals connecting vertices of the polygon Σ. The corners of each triangle are stored locally, only at the nodes that are inside this triangle, along with the corners of the (at most 3) triangles that are adjacent to it. The dual graph of the triangulation is a planar graph $\mathcal{D}$. By removing cut edges, one per hole from $\mathcal{D}$, we obtain a tree T. We then embed the tree in the hyperbolic plane, such that by tiling copies of the tree we obtain an infinite repeating tree $\mathcal{T}$.

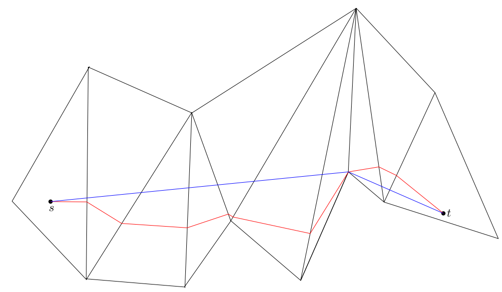

Fig. 2. The solid path is the optimal path; the dashed path is the greedy path.

We use a two-level structure in our scheme.

Level 1: Using a similar idea as in [1], we can use a greedy algorithm to find a path in the universal covering space of $\mathcal{D}$: the tree $\mathcal{T}$, with only knowledge of the triangles that contain the source and the destination. This top-level greedy algorithm reveals a sequence of triangles that contains the shortest path of the required homotopy type.

Level 2: Now we have a sequence, $\Delta = \{\triangle_1, \triangle_2, \cdots, \triangle_n\}$, of triangles Δ_i, with adjacent triangles in the sequence sharing a common edge. We develop a greedy, local algorithm that "navigates" inside the triangles with total travel length at most a constant times the shortest path inside Δ. The idea is to move through the sequence greedily, always taking the shortest path to the boundary of the next triangle. This local algorithm does not need to know the entire sequence of triangles but only the current triangle and the shared diagonal with the next triangle. The following theorem is a major technical contribution of this paper.

Theorem 1. *The length of the greedy path is at most* $15\pi + 2$ *times the length of the shortest path.*

Our low-level greedy routing algorithm within a sequence of triangles can be extended to greedy routing inside a sequence of consecutively adjacent simple polygons, with the same worst-case stretch, as we can always further triangulate each simple

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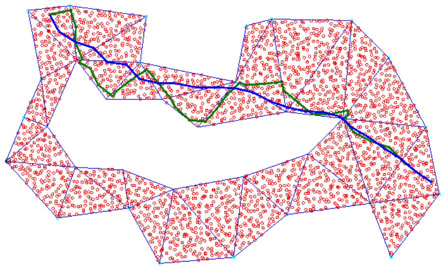
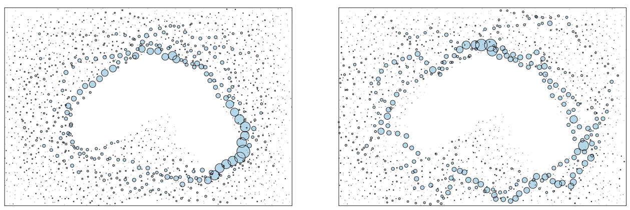


Fig. 3. The blue line is shortest path; the green line is the path used by our algorithm.



(a) Shortest Path (b) Our algorithm

Fig. 4. In this figure, each node is represented by a circle, and the diameter of circle is proportional to the traffic load at that node. For shortest path in 4(a), the loads are largely around the boundary. Our Greedy routing method 4(b) gives better load balance.

polygon. Therefore, as a bonus feature, our new algorithm can replace the local routing scheme in the previous geometric routing schemes that use network decompositions [3]–[5] and provide constant stretch in the local routing part.

Here are the simulations of our algorithm in terms of the routing stretch and load balancing. From an evaluation point of view, we are interested in the performance of the Level 2 of the algorithm.

Fig. 3 is a polygon domain with a hole. The stretch is 1.34.

Fig. 4 shows the traffic load distribution of the shortest path algorithm and our algorithm.

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