Problem 6.6.2 Solution

Knowing that the probability that a voice call occurs is 0.8 and the probability that a data call occurs is 0.2 we can define the random variable \( D_i \) as the number of data calls in a single telephone call. It is obvious that for any \( i \) there are only two possible values for \( D_i \), namely 0 and 1. Furthermore for all \( i \) the \( D_i \)'s are independent and identically distributed with the following PMF.

\[
P_D (d) = \begin{cases} 
0.8 & d = 0 \\
0.2 & d = 1 \\
0 & \text{otherwise}
\end{cases}
\]  

From the above we can determine that

\[
E[D] = 0.2 \quad \text{Var}[D] = 0.2 - 0.04 = 0.16
\]  

With these facts, we can answer the questions posed by the problem.

(a) \( E[K_{100}] = 100E[D] = 20 \)

(b) \( \text{Var}[K_{100}] = \sqrt{100 \text{Var}[D]} = \sqrt{4} = 4 \)

(c) \( P[K_{100} \geq 18] = 1 - \Phi\left(\frac{18 - 20}{4}\right) = 1 - \Phi(-1/2) = \Phi(1/2) = 0.6915 \)

(d) \( P[16 \leq K_{100} \leq 24] = \Phi\left(\frac{24 - 20}{4}\right) - \Phi\left(\frac{16 - 20}{4}\right) = \Phi(1) - \Phi(-1) = 2\Phi(1) - 1 = 0.6826 \)

Problem 6.7.1 Solution

In Problem 6.2.6, we learned that a sum of iid Poisson random variables is a Poisson random variable. Hence \( W_n \) is a Poisson random variable with mean \( E[W_n] = nE[K] = n \). Thus \( W_n \) has variance \( \text{Var}[W_n] = n \) and PMF

\[
P_{W_n} (w) = \begin{cases} 
\frac{n^w e^{-n}}{w!} & w = 0, 1, 2, \ldots \\
0 & \text{otherwise}
\end{cases} 
\]  

All of this implies that we can exactly calculate

\[
P[W_n = n] = P_{W_n} (n) = \frac{n^ne^{-n}}{n!}
\]  

Since we can perform the exact calculation, using a central limit theorem may seem silly; however for large \( n \), calculating \( n^n \) or \( n! \) is difficult for large \( n \). Moreover, it’s interesting to see how good the approximation is. In this case, the approximation is

\[
P[W_n = n] \approx \Phi\left(\frac{n + 0.5 - n}{\sqrt{n}}\right) - \Phi\left(\frac{n - 0.5 - n}{\sqrt{n}}\right) = 2\Phi\left(\frac{1}{2\sqrt{n}}\right) - 1
\]  

The comparison of the exact calculation and the approximation are given in the following table.

<table>
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<tr>
<th>( n )</th>
<th>Exact</th>
<th>Approximate</th>
</tr>
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<td>64</td>
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